Attractor in non-conformal boost-invariant plasmas

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Introduction

Hydrodynamics is an effective macroscopic theory describing long wavelength excitations in a fluid, and is expected to break down in systems with large spatial or temporal gradients. Recently, several studies comparing results of higher-order hydrodynamic theories to conformal kinetic theories in boost-invariant flow profiles have revealed a surprising success of hydrodynamics in providing a near-accurate description of the system's macroscopic dynamics even when the medium is very far from local equilibrium. A key feature that emerged from these studies is that hydrodynamics is governed by a far-from-equilibrium attractor [1] to which different initializations of normalized dissipative quantities decay via power law at high Knudsen-numbers over a time-scale controlled by the relaxation-time [2, 3].

However, almost all of these comparisons of hydrodynamics with kinetic theory have focused on conformal systems. In the present study, we investigate the domain of applicability of second-order *non-conformal* hydrodynamics by comparing it with kinetic theory for systems of massive particles undergoing (0+1) dimensional expansion with Bjorken symmetry [4, 5].

Bjorken flow is conveniently described in Milne coordinates, with proper time $\tau \equiv \sqrt{t^2 - z^2}$ (where t is time and z the longitudinal Cartesian coordinate), space-time rapidity $\eta \equiv \tanh^{-1}(z/t)$, and metric $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -\tau^2)$. In these coordinates the flow appears static, $u^\mu = (1, \vec{0})$, and all macroscopic quantities only depend on the proper time. Bjorken symmetry dictates the energy momentum tensor to be diagonal, $T^{\mu\nu} = \mathrm{diag}(\epsilon, P_T, P_T, P_L)$, where ϵ, P_T and P_L are the energy density, effective transverse and longitudinal pressures respectively. The pressures are expressed in terms of the equilibrium pressure (P), bulk pressure (Π), and the single independent component of the shear stress tensor $\pi \equiv -\tau^2 \pi^{\eta\eta}$ as $P_T = P + \Pi + \pi/2$ and $P_L = P + \Pi - \pi$.

We shall consider the evolution of single particle distribution function $f(x,p)$ to be governed by the Boltzmann equation with a collision term in the relaxation time approximation (RTA): $\partial_\tau f = -(f - f_{\mathrm{eq}})/\tau_R(\tau)$. Here f_{eq} represents the single particle equilibrium distribution function with Boltzmann statistics. The relaxation time τ_R sets the timescale for equilibration and is parametrized as $\tau_R = 5C/T$ where T is the temperature and C is a unitless constant. The above equation can be solved exactly [6], and appropriate moments of the distribution function gives exact evolution of hydrodynamic quantities. The initial distribution function is considered to have a form which allows for large bulk and shear stresses at initial time τ_0 [4, 5].

Results

We impose initial conditions at $\tau_0 = 0.1\,\mathrm{fm}/c$ with initial temperature $T_0 = 500\,\mathrm{MeV}$. We take $m = 50\,\mathrm{MeV}$ for the particle mass such that m/T is initially small and the fluid's equation of state and transport coefficients are close to their conformal limits at early times. In all figures, solid curves are solutions of the RTA Boltzmann equation and dashed ones represents solutions of second-order Chapman-Enskog hydrodynamics [7]. Dotted curve is the first-order Navier-

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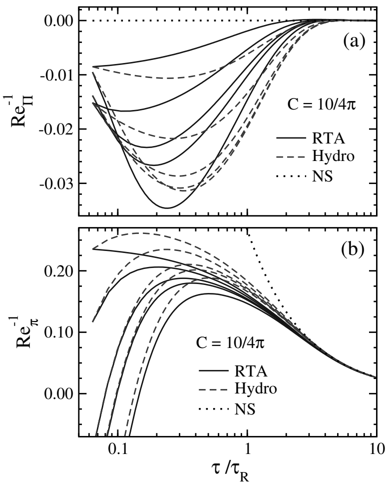

FIG. 1: Scaled time evolution of the (a) bulk and (b) shear inverse Reynolds numbers.

Stokes (NS) solution with $C = 10/4\pi$.

For small bulk viscous pressures one naively expects to recover the known attractor structure of conformal systems. Figure 1 shows the evolution of the bulk and shear inverse Reynolds numbers for such initial conditions; $\mathrm{Re}_\Pi^{-1} \equiv \Pi/(\epsilon{+}P)$ and $\mathrm{Re}_\pi^{-1} \equiv \pi/(\epsilon{+}P)$, as functions of the scaled time $\bar{\tau} \equiv \tau/\tau_R$ (for $C = 10/4\pi$). However, the shear stress trajectories shown in Fig. 1b are seen to converge only at $\bar{\tau} \approx 2$ (magnitudes of initial normalized bulk viscous pressure are small as can be seen in Fig. 1a). This strongly contrasts with the pattern observed in conformal systems where trajectories with different initializations of normalized shear stresses rapidly approach an early-time attractor on a much shorter time scale controlled by the initialization time τ_0 [2, 3]. Large initial bulk stress further delays the convergence of trajectories [4, 5]. Therefore, we conclude that there is no evidence of an early-time attractor for non-conformal fluid in shear or bulk stresses.

Figure 2 shows the evolution of the scaled longitudinal pressure P_L/P as a function of the scaled time $\bar{\tau}$. The initial conditions considered correspond to those used in Fig. 1. In addition, we also initialize with vanishing shear stress and large bulk stress, as well as with different value of the parameter $C = 3/4\pi$. All kinetic theory solutions are seen

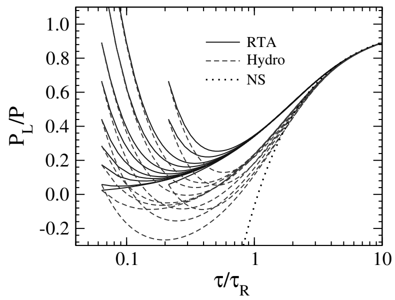

FIG. 2: Evolution of the scaled longitudinal pressure.

to join a universal attractor that starts from $P_L/P \approx 0$ at $\bar{\tau}_0 \to 0$ at early times $\bar{\tau} \lesssim 0.5$. As the system begins to isotropize, this universal curve joins the first-order hydrodynamic NS attractor at $\bar{\tau} \gtrsim 4$ and approaches unity. Therefore, only $P_L = P{+}\Pi{-}\pi$ (a combination of Π and π) shows an early-time attractor behavior, driven by the strong longitudinal flow at early times. We do not expect similar early-time attractors in systems where the early-time dynamics is not dominated by free-streaming. The hydrodynamic trajectories in Fig. 2 do not exhibit an early-time attractor; universality is only seen at $\bar{\tau} \gtrsim 4$ after they join the NS attractor. Clearly, second-order non-conformal hydrodynamics does not provide a very accurate approximation to the underlying kinetic theory when $\bar{\tau} < 3$.

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