Electric charge transport in QCD medium in the presence of time-varying electromagnetic fields

Gowthama K K,* Manu Kurian, and Vinod Chandra
Indian Institute of Technology Gandhinagar, Gandhinagar-382355, Gujarat, India

Introduction

Heavy-ion collision experiments at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) have confirmed the existence of nuclear matter-Quark Gluon plasma (QGP). It is widely believed that intense magnetic fields are created, especially in the initial stages of collision experiments. However, a conclusive model to describe the evolution of the magnetic field in the medium is yet to be known. In this context, the study of the QCD medium response to the external electromagnetic fields is relevant and may have a dependence on the charge asymmetric flow at the RHIC and LHC and other interesting aspects such as Chiral Magnetic Effect and magnetohydrodynamics. The electric charge transport in a magnetized medium can be studied in terms of induced electric and Hall current densities and associated conductivities. Here, we have analyzed three aspects of the charge transport in the QCD medium, namely (i) QCD medium response to the time-varying external electric field [1], (ii) set-up a general formalism to explore the electric charge transport in the presence of time-varying electromagnetic fields [2], (iii) medium response in an anisotropic QCD medium [2]. To that end, we have estimated the general form of quark degrees of freedom in the presence of time-dependent fields within the framework of kinetic theory for both isotropic and anisotropic cases.

Formalism

The induced vector current in the QCD medium with a finite quark chemical potential μ can be defined as,

$$j^i = 2N_c \sum_f \int d\frac{d^3\mathbf{p}}{(2\pi)^3} v^i \Big(q_q f_q - q_{\bar{q}} f_{\bar{q}} \Big), \quad (1)$$

where $f_k = f_k^0 + \delta f_k$ (the subscript k incidates the particle species) is the quark/antiquark distribution function, v_i velocity component, and N_c is number of colors. We chose the following ansatz for non-equilibrium part of the distribution function δf_k due to the inhomogeneous fields,

$$\delta f_k = (\mathbf{p}.\mathbf{\Xi}) \frac{\partial f_k^0}{\partial \epsilon}, \quad (2)$$

where the vector $\mathbf{\Xi}$ is related to the strength of electric and magnetic fields and its derivatives and has the form as follows,

$$\mathbf{\Xi} = \alpha_1 \mathbf{E} + \alpha_2 \dot{\mathbf{E}} + \alpha_3 (\mathbf{E} \times \mathbf{B}) + \alpha_4 (\dot{\mathbf{E}} \times \mathbf{B}) + \alpha_5 (\mathbf{E} \times \dot{\mathbf{B}}) + \alpha_6 (\mathbf{\nabla} \times \mathbf{E}) + \alpha_7 \mathbf{B} + \alpha_8 \dot{\mathbf{B}} + \alpha_9 (\mathbf{\nabla} \times \mathbf{B}),$$

with α_i ($i = (1, 2, .., 9)$) are the unknown functions that relate to the respective transport coefficients associated with the electric charge transport and can be estimated by employing the relativistic transport equation. The transport equation that describes the dynamics of the momentum distribution function can be defined as,

$$p^\mu \, \partial_\mu f_k(x,p) + q_{f_k} F^{\mu\nu} p_\nu \partial_\mu^{(p)} f_k = -(u.p) \frac{\delta f_k}{\tau_R},$$

where u^μ is the fluid velocity, $F^{\mu\nu}$ is the field strength tensor, and τ_R is the relaxation time. We have compute the α's for various cases of electromagnetic fields by solving the Boltzmann equation.

I. Constant electromagnetic fields

When the electric and magnetic fields are constants, we obtain α_1 and α_3 as,

$$\alpha_1 = \frac{-\epsilon q_{f_k}}{\tau_R [(\frac{\epsilon}{\tau_R})^2 + (q_{f_k} B)^2]}, \quad \alpha_3 = \frac{-q_{f_k}^2}{[(\frac{\epsilon}{\tau_R})^2 + (q_{f_k} B)^2]}.$$

*Electronic address: `k$_$gowthama@iitgn.ac.in`

II. Medium response to time-varying electric field

For the case with time-varying electric fields and constant magnetic fields we get the same form of α_1 and α_3 with new terms in α_2 andα_4, given by [1],

$$\alpha_2 = \frac{q_{f_k}\epsilon[(\frac{\epsilon}{\tau_R})^2 - (q_{f_k}B)^2]}{[(\frac{\epsilon}{\tau_R})^2 + (q_{f_k}B)^2]^2}, \quad \alpha_4 = \frac{2q_{f_k}^2\epsilon^2}{\tau_R[(\frac{\epsilon}{\tau_R})^2 + (q_{f_k}B)^2]^2}.$$

III: Time-varying electromagnetic fields

In the presence of time-varying electromagnetic fields, the current density take the form as $\mathbf{j} = j_e\hat{\mathbf{e}} + j_H(\hat{\mathbf{e}} \times \hat{\mathbf{b}})$ with,

$$j_e = j_e^{(0)} + j_e^{(1)}, \quad j_H = j_H^{(0)} + j_H^{(1)} + j_H^{(2)}, \quad (3)$$

where j_e corresponds to the electric current in the direction of the electric field $\hat{\mathbf{e}}$ and j_H is the electrical current in the direction perpendicular to both electric and magnetic fields $(\hat{\mathbf{e}} \times \hat{\mathbf{b}})$ as given in [2].

Effects of momentum anisotropy

Momentum anisotropy can be model by compressing or expanding the isotropic distribution function f_k^0 as,

$$f_{(\text{aniso})_k} = \sqrt{1+\xi}\, f_k^0\left(\sqrt{p^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right), \quad (4)$$

The electric and Hall currents develop additional components due to the anisotropy as [2],

$$(j_e)_{\text{aniso}} = j_e^{(0)} + \delta j_e^{(0)} + j_e^{(1)} + \delta j_e^{(1)},$$
$$(j_H)_{\text{aniso}} = j_H^{(0)} + \delta j_H^{(0)} + j_H^{(1)} + \delta j_H^{(1)} + j_H^{(2)} + \delta j_H^{(2)}.$$

The forms of the additional components are described in detail in Ref [2].

Results and observations

To analyse the impact of time-dependence of the electromagnetic fields to the QCD medium response, we define the following ratios,

$$R_e = \frac{j_e^{(0)}}{ET} + \frac{j_e^{(1)}}{ET}, \quad R_H = \frac{j_H^{(0)}}{EBT} + \frac{j_H^{(1)}}{EBT} + \frac{j_H^{(2)}}{EBT}.$$

For the case of constant electromagnetic fields, the term $j_e^{(0)}/(ET) = \sigma_e/T$ and the term $j_e^{(1)}/(ET)$ denotes the correction due to the time dependence of the electric field. In the similar way, $j_H^{(0)}$ describes the leading order term and the quantities $j_H^{(1)}$ and $j_H^{(2)}$ denote the corrections to the current density in the direction $(\hat{\mathbf{e}} \times \hat{\mathbf{b}})$. Note that the strength of the time dependence of the fields is quantified in terms of decay time. We have depicted the effect of the inhomogeneity of time of the external electromagnetic fields on the temperature dependence of R_e in Fig. 1. It is seen that the time dependence of the fields has a visible impact on the QCD medium response. For the case of a time-varying electric field and a constant magnetic field, the value of R_e is higher in comparison with the case of constant fields due to the additional contribution from $\dot{\mathbf{E}}$. The inclusion of time inhomogeneity of the magnetic field further introduces back current in the QGP medium. The same observation holds true for R_e. In addition, we have observed that $j_e\hat{\mathbf{e}}$ and $j_H(\hat{\mathbf{e}} \times \hat{\mathbf{b}})$ decrease with an increase in momentum anisotropy of the medium. The additional components to current densities may perhaps play a vital role in the magnetohydrodynamical framework for the QCD medium in the collision experiments.

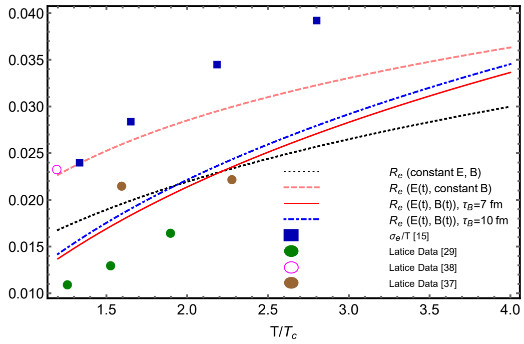

FIG. 1: The temperature behaviour of R_e for various choices of external fields. The results are compared with lattice data, Ref. [2] for details.

References

[1] K.K.Gowthama, M. Kurian and V. Chandra, Phys. Rev. D **103**, 074017 (2021).

[2] K.K.Gowthama, M. Kurian and V. Chandra, arXiv:2108.06791 (2021) [hep-ph].