Study of Deuteron Ground state using Quantum Hamilton-Jacobi Theory

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Introduction

In this paper, we discuss the fundamental problem in nuclear physics, the deuteron using Quantum Hamilton Jacobi Theory[1]. This formalism is an extension of the Bohmian mechanics in which both position and momentum variables are considered to be complex valued. Other important aspect is that the wavefunction is written in polar form as function of action. This results in an intrinsic quantum potential(QP) and gives rise to concept of quantum trajectories. It has been already shown[2] that the Hydrogen atom's shell structure and stablity have been possible due to the attractive Coulomb force being balanced by the repulsive quantum force arising from the intrinsic QP. Tracing the complex eigen-trajectories of electron due to various quantum numbers results in corresponding exact eigen values at every point on the eigen-trajectory, this confirming the stationary state concept. Our premise in this paper is, that the inter-nucleon potential in np interaction is similarly the resultant of external central potential of Yukawa type and the intrinsic QP. *Repulsive core* at very small inter-nucleon distance and *charge radius* of deuteron can be explained with the help of total modified potential.

Morphed Gravitational Potential Energy

We choose *Morphed Gravitational Potential Energy* (MGPE)[3] as the agent of nuclear interaction because it resembles with Yukawa potential and it's time independent Schrodinger equation has analytical solution.

$$V(r) = -\frac{g^2\hbar c}{M_0^2}\frac{m_p m_n}{r} \tag{1}$$

$g^2 = 0.032384$ & $M_0 = 542.9876336\ Mev/c^2$ for deuteron. m_p & m_n are masses of proton and neutron respectively.
The radial wave function is given by

$$R_{nl} = Ae^{\frac{-\rho}{2}}\rho^l L_{n+l}^{2l+1}(\rho) \tag{2}$$

with $\rho = \frac{2r}{na_0}$ and $a_0 = 4.31734\ fm$ is the distance between the two nucleons.

Quantum Potential

The *Quantum Hamilton-Jacobi* equation[1]

$$\frac{\partial S(t,\mathbf{q})}{\partial t} + \left[\frac{\mathbf{p}^2}{2m} + V(t,\mathbf{q}) + V_Q\right] = 0 \tag{3}$$

is derived by substituting

$$\Psi(t,\mathbf{q}) = e^{\frac{iS(t,\mathbf{q})}{\hbar}}$$

into the Time Dependent Schrödinger equation. One can rewrite the QHJ equation as

$$\frac{\partial S(t,\mathbf{q})}{\partial t} + H_Q = 0 \tag{4}$$

where H_Q is quantum Hamiltonian.The canonical variable $\mathbf{p}$ from Hamilton theory is given by

$$\mathbf{p} = \nabla S = -i\hbar\nabla \ln \Psi(t,\mathbf{q})$$

The Quantum Potential,

$$V_Q = \frac{\hbar}{2mi}\nabla.\mathbf{p} = -\frac{\hbar^2}{2m}\nabla^2 \ln \Psi(t,\mathbf{q}) \tag{5}$$

is the potential which is intrinsic to all quantum systems.

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Modified Potential of Deuteron

By describing Laplacian (∇^2) in spherical polar coordinates[2], we can obtain quantum potential in $3D$. However, since nuclear interaction is central in nature, only radial part is significant in our case.The total radial potential will be given by

$$V_T = \frac{\hbar^2}{8mr^2}(4+cot^2\theta) - \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2}\ln(R_{nl})\right) + V(r) \quad (6)$$

Ground state

Due to non zero quadruple moment and even parity of deuteron, it's ground state is a combination of ${}^3S_1$ and ${}^3D_1$ state.

$$\psi_{deutron} = a_s\psi_s + a_d\psi_d \quad (7)$$

The values of a_S and a_d are determined as[3]

$$a_s = 0.999893,\ a_d = 0.014629$$

Using Eq. 2, one can find radial wave functions for S and D states. Using Eq. 6, one obtains V_T. The total potential is expressed in dimensionless form by dividing V_T by $\hbar^2/2ma_0^2$.

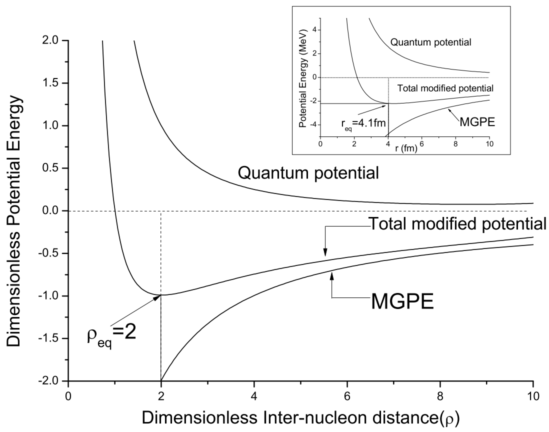

FIG. 1: Radial variation of potential in 3S1 state

Complex trajectory of Deuteron

The motion of deuteron in complex plane is given by Hamilton's equation

$$\dot{q}_i = \frac{\partial H_Q}{\partial p_i} \quad (8)$$

Therefore, for S-state, with $\tau = t\hbar/2ma_0^2$

$$\frac{\partial\rho}{\partial\tau} = -4i\frac{2-\rho}{\rho} \quad (9)$$

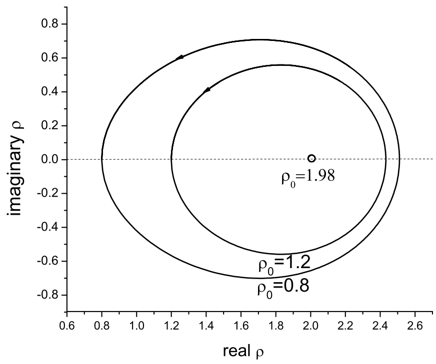

FIG. 2: Complex trajectory of Deuteron for various initial positions.

Results and Discussion

Following inferences can be made:
1.Equilibrium point $r_{eq} = 4.1\ fm$ is the point where the modified potential has maximum depth and deuteron is most stable.
2.The potential energy corresponding to equilibrium point corresponds to binding energy of deuteron, i.e.-2.224 MeV.
3.A repulsive core appears for $r < r_{eq}$, i.e. at very small inter-nucleon distance.
4.Half of r_{eq} gives stability radius which we propose to be charge radius (2.05 fm).
5.The deuteron forms closed trajectories around equilibrium point($\rho = 2$).

References

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[2] C. D. Yang, "Quantum dynamics of hydrogen atom in complex space", Ann of Phys., (319), 399- 443, 2005.
[3] Chandraraju, Chintalapati. (2013). Theory of Deuteron and MCPE. J