Isospin mixing in self-conjugate nuclei: A statistical approach using forbidden E1 transitions

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Introduction

The concept of isospin was put forward by Heisenberg to describe the charge independence property of nuclear forces, and a detailed theoretical framework of the same was provided by Wigner [1]. The isospin quantum number T treats protons and neutrons on the same footing and puts strict selection rules on nuclear reactions, decays, and transitions [2]. These selection rules have broad implications for nuclear structure. However, isospin symmetry is not strictly conserved due to Coulomb and other isospin symmetry-breaking effects [3]. As a result, the extreme frontiers of isospin symmetry conservation and the (in)validity of isospin selection rules therein have received wider attention from modern nuclear physicists in search of new and exciting physics. For example, the isospin selection rules for super-allowed β-decay can be used to test the unitarity of the Cabibbo-Kobayashi-Maskawa matrix, which in turn can be used to connect strong force with the electroweak force [4]. These studies require precise knowledge of the nature, extent, and effects of the validity of isospin symmetry. As a result, isospin symmetry-breaking or isospin mixing has been the subject of many studies in recent years, including β-decay and γ-decay studies.

In the present work, I have explored the possibility of determining the degree of isospin mixing in self-conjugate (nuclei having an equal number of protons and neutrons) nuclei using selection rules for the electric dipole transitions. ΔT=0 E1 transitions are forbidden in self-conjugate nuclei by the isospin selection rules [2]. However, these E1 transitions are readily observed experimentally, even when the low-lying states have the same isospin T. Although, they are rather retarded (at least an order of magnitude on the average) with respect to the allowed ones in the neighboring nuclei. The experimental observation of these forbidden E1 transitions in self-conjugate nuclei may be explained by considering the excited states between which the transition takes place to be of mixed isospins. This opens channels other than the ΔT=0 channel for the transition to proceed, its transition strength determined by the degree of isospin mixing.

An average value of the experimentally measured transition strengths of these allowed and forbidden E1 transitions can provide a measure of the isospin mixing amplitudes in these nuclei. In this work, I have adopted a statistical approach devised by Prof. D. H. Wilkinson in 1958 [5]. The same methodology was employed rather spasmodically over the years by other workers for a handful of other nuclei [6], but a systematic study has not been carried out for any series of nuclei since then, as per the author's knowledge. Therefore, it naturally seemed to be an interesting and worthwhile study to revisit Prof. Wilkinson's statistical formalism to understand the variation of isospin mixing across self-conjugate nuclei. Thus, a systematic study of the variation of isospin mixing in self-conjugate nuclei is the subject of the present study.

Formalism

It is well known that the charge-dependent parts of the nuclear interaction (e.g., the Coulomb interaction) violate isospin symmetry, which leads to states of mixed isospin. For the present purpose, the wave function of the parent and daughter levels between which the forbidden E1 transition takes place can be written as [5],

$$\Psi = \Psi(T) + \sum_{T'} \frac{H^C_{T'T}}{E_T - E_{T'}} \Psi(T') = \Psi(T) + \sum_{T'} \alpha_T \Psi(T')$$

where $H^C{}_{T'T}$ is the matrix element of the charge-dependent Hamiltonian between the states $\psi(T)$ and $\psi(T')$ with energies E_T and $E_{T'}$, respectively. The coefficient $\alpha_T\,(T')$ squared is known as the isospin mixing probability that determines the proportion of impurity of state $\psi(T')$ into state $\psi(T)$. It is defined as the ratio of the average square of the matrix element of the forbidden transitions to the average square of the matrix element of the allowed transitions [5], i.e.

$$\alpha_T{}^2(\mathrm{T'}) = \frac{\overline{|M|^2}(E1)_{\Delta T=0}}{\overline{|M|^2}(E1)_{\Delta T=1}}$$

Calculating the isospin mixing in this way is purely statistical because when averaging, the dependence of transition probabilities on quantum numbers other than isospin is ignored. Knowing $\alpha_T(T')$, one can determine the average value of the mixing matrix element from the following expression,

$$H^C_{T'T} = \alpha_T(T')\,\Delta_{T'T}$$

where $\Delta_{T'T}$ is the average energy difference between levels with isospins T' and T.

Results and Discussions

Based on the above formalism, the isospin mixing values, $\alpha^2_T(T')$ are calculated for five *sd*-shell nuclei. The matrix elements of the allowed and forbidden $E1$ transitions were calculated from their transition strengths, which were in turn, taken from NNDC [7] and cross-references therein. The results are tabulated in Table 1, where the calculated values of $\Delta_{T'T}$ and $H^C{}_{T'T}$ are also listed.

Table 1: Values of isospin mixing probabilities, average energy differences between T and T' levels, and the mixing matrix elements for a few *sd*-shell nuclei.

Nucleus	$\alpha^2_T(T')$	$\Delta_{T'T}$ (keV)	$H^C{}_{T'T}$ (keV)
^{24}Mg	0.5	60	40
^{26}Al	0.2	130	60
^{28}Si	0.09	40	12
^{30}P	0.12	180	60
^{32}S	0.07	190	50

From the table, it is seen that the isospin mixing probabilities and the mixing matrix elements are in agreement with those expected from the literature.

However, it has to be borne in mind that any statistical analysis is considered meaningful only if it has been undertaken on data of high quality. Thus, for the present study, this puts a stringent requirement of the knowledge of a complete level scheme- with unambiguous spin, parity, and isospin assisgnment of levels and accurately measured transition strengths. Having said that, even with the recent unprecedented advancements in the field of gamma-ray spectroscopy, complete spectroscopy could be successfully performed till now for only ^{26}Al and ^{30}P nuclei [7]. Thus, although the present study may not, by any measure, be considered an accurate representation or a complete description statistically, it definitely serves as a step towards understanding some lesser understood statistical problems in nuclear structure, like the effects of broken isospin symmetry on eigenvalue and transition strength distributions, which are potentially sensitive tools in the test of the random matrix theory.

The present study shall be extended to all possible self-conjugate nuclei.

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