The energy gap between $1h_{11/2}$ and $1g_{7/2}$ proton single particle levels in Sn-isotopes and tensor force

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Introduction

Some of the noteworthy phenomena for which the model calculations have used the tensor interaction to explain the data are, the $2p_{3/2}$ and $1f_{5/2}$ proton single particle (s.p.) levels crossing in Ni-isotopes, $2s_{1/2}$ and $1d_{3/2}$ proton s.p. levels crossing in Ca-isotopes, the $1h_{11/2}$-$1g_{7/2}$ s.p. proton energy gap along the Sn and Sb-isotopic chains, the $1i_{13/2}$-$1h_{9/2}$ s.p. neutron energy gap in isotones of N=82, etc. [1]. The finite range simple effective interaction (SEI), we use in our model calculations, however, explain some of these features quantitatively at the mean-field+BCS level, but could not produce the $1h_{11/2}$ and $1g_{7/2}$ s.p. proton levels splitting along the Sn-isotopic chain as well as the $1i_{13/2}$ and $1h_{9/2}$ s.p. neutron levels splitting in N=82 isotones [2]. To our knowledge, there is no model calculation performed till date that could explain these two features of Z=50 isotopes and N=82 isotones quantitatively over the whole range of mass number A. We have included a short range tensor part [3], and searched for the modification in the mean-field level in order to reproduce the $1h_{11/2}$-$1g_{7/2}$ s.p. proton data in Z=50 isotopes in a quantitative way.

Formalism

The SEI introduced in Ref.[4] by Behera and collaborators, contains altogether 11-numbers of parameters in nuclear matter (NM), namely, α, γ, b, x_0, x_3, t_0, t_3, W, B, H, and M is given by,

$$\begin{aligned} V_{eff} &= t_0(1+x_0P_\sigma)\delta(\vec{r}) \\ &+ \frac{t_3}{6}(1+x_3P_\sigma)\left(\frac{\rho(\vec{R})}{1+b\rho(\vec{R})}\right)^\gamma \delta(\vec{r}) \\ &+ (W+BP_\sigma - HP_\tau - MP_\sigma P_\tau)f(\vec{r}) \\ &+ \text{Spin-orbit part.} \end{aligned} \quad (1)$$

The Skyrme-type spin-orbit interaction, and a short-range tensor force having triplet-even and triplet-odd terms [3], with strengths T and U, respectively, is used for which the total spin-orbit potential becomes,

$$\mathbf{V}^q_{SO} = \frac{W_0}{2}\left(2\nabla\rho_q + \nabla\rho_{q'}\right) + \alpha_T \mathbf{J}_q + \beta_T \mathbf{J}_{q'}, \quad (2)$$

$$\text{with} \quad \alpha_T = \frac{5}{12}U \qquad \beta_T = \frac{5}{24}(T+U), \quad (3)$$

where, $q(q')$=n (p), $\rho_{q(q')}$ and $J_{q(q')}$ are respective nucleon and spin densities, and W_0 is the spin-orbit strength parameter. The finite nuclei calculations has been done in the so-called Quasi-local Density Functional Theory (QLDFT) using BCS pairing.

Results and Discussion

In the framework of QLDFT, the SEI at mean-field+BCS level could reproduce the $2p_{3/2}$ and $1f_{5/2}$ proton s.p. levels crossing in Ni-isotopes and magic character of ^{78}Ni, spin-parity inversion in Cu-isotopes at right mass number, the $2s_{1/2}$ and $1d_{3/2}$ proton s.p. levels crossing in Ca-isotopes, the kink features in Pb-isotopes at N=126 [5]. But, it could not predict the experimental variations in the $1h_{11/2} - 1g_{7/2}$ proton s.p. energy gap in the Z=50 and the $1i_{13/2} - 1h_{9/2}$ s.p. neutron energy gap in the N=82 isotonic chain [2]. In both the cases, a linear dependence is obtained having relatively wider separation between the states as shown by curve-(a) in panels (I) and (II) of Fig. 1. By including the tensor interaction the results in Z=50 isotopes and N=82 isotones could be reproduced qualitatively as can be seen from the curves marked-(b) in panel (I) and (II) of Figs.1, respectively. The tensor interaction modifies the spin-orbit splitting of the s.p. levels that together with the pairing interaction could produce experimental oscillatory trend, but it could not modify the positioning of the s.p. levels to reproduce the data quantitatively. This facts points to the crucial necessity for modification

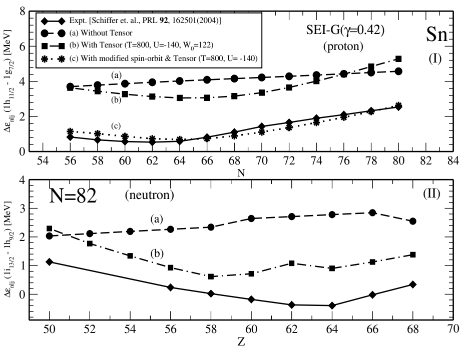

FIG. 1: (a)Energy differences between $1h_{11/2}$ and $1g_{7/2}$ proton s.p. levels in Sn isotopes, (b) Energy differences between $1i_{13/2}$ and $1h_{9/2}$ neutron s.p. levels in N = 82 computed with and without tensor interaction. The experimental data are taken from Ref.[2]

at the mean-field level. Following the concept of Reinhard and Flocard [6], where the total spin-orbit contribution is split into the isoscalar and isovector parts but with different strength factors κ and κ', used as adjustable parameters. For κ=κ', we recover the normal spin-orbit strength parameter W_0.

$$\mathbf{V^q_{SO}} = -\Big[(\kappa+\kappa')\nabla\rho + \kappa'\nabla\rho_\mathbf{q} + \alpha_\mathbf{T}\mathbf{J_q} + \beta_\mathbf{T}\mathbf{J_{q'}}\Big]. \quad (4)$$

For the same tensor interaction, with the values of (κ+κ')=120 MeV fm^5 and κ'=-40 MeV fm^5, the proton $1h_{11/2} - 1g_{7/2}$ s.p. gap in Z=50 isotopes could be reproduced in close proximity with the experimental data over the whole mass region, as shown by the curve-(c) in panels (I) of Fig.1. The requirement for the purpose in terms of the positioning of neutron s.p. levels and occupation probability has been examined in the two panels of Fig.2. The $2d_{5/2}$ neutron level need to be less bound, whereas, the $1h_{11/2}$ neutron level requires to be more bound lying close to the $2d_{3/2}$ neutron level which has now moved-up. The occupation probability of the $2d_{5/2}$ neutron level is enhanced, whereas, there is a decrease in the occupation probability of $1g_{7/2}$ neutron level in the range N=50-64. The occupancies of $1h_{11/2}$ and $2d_{3/2}$ neutron levels are similar over the range of the neutron number. All these changes in s.p. neutron levels and

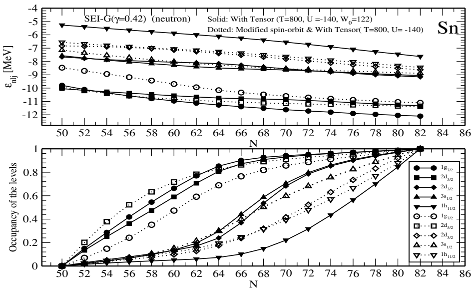

FIG. 2: (a) Neutron s.p energy levels and (b)Occupation probability of the neutron levels of Sn isotopes in the N = 50 to N = 82 major shell.

their occupancies are relative to their respective predictions of the normal spin-orbit case shown in the same Fig.2.

Conclusion

The tensor interaction plays crucial role in the interpretation of the features observed in the finite nuclei under the extreme condition of isospin asymmetry. But at the same time, the mean-field resulting from the interaction that determines the position and occupancy of the single particle energy states, need to be ascertained as close as to the actual for the description of the phenomena associated to the exotic nuclei.

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