Bandhead Energies of K=1/2 bands of Three Quasiparticle Quadruplets

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Introduction

In a deformed axially symmetric nucleus, the coupling of three particles having projection quantum number K_1, K_2, K_3 leads to a three-quasiparticle (3QP) quadruplet with the resultant $K = |K_1 \pm K_2 \pm K_3|$. These four intrinsic states split-up due to the residual neutron-proton (*n-p*) interactions among three nucleons. In order to estimate the bandhead energies of the above said four intrinsic 3QP states, Jain and Jain proposed an empirical model [1] but in this model the diagonal contributions of particle-particle couplings (ppc), rotor particle couplings (rpc) and irrotational contributions appearing from the interactions of valence particles were ignored. In this paper, we presented the results of bandhead energies of K=1/2 members of 3qp quadruplets observed in ^{175}Yb and ^{177}Ta and compared with experimental data.

The Model

The present model formulation is an extension earlier model developed by K. Jain *et al.* [1]. In present calculations, the bandhead energies is obtained by including the contributions arising from rotor-particle and particle-particle coupling and irrotational motion of valence protons/neutrons which were neglected in the earlier formulation [1].

The bandhead energy of particular member of a 3QP quadruplet can be written as [3]:

$$E(K) = E_{qp} + E_{rot.} + E_{irrot.} + E_{resi.} + E_{rpc} + E_{ppc} + E_{pair}$$

where, total quasiparticle (E_{qp}) energy is the sum of the one quasiparticle energy of individual nucleons. The rotational energy (E_{rot}) is given by:

$$E_{rot} = \frac{\hbar^2}{2\Im}(I(I+1) - K^2)$$

E_{pair} is proton and neutron pairing energy and can be obtained from experimental data of binding energies [2] using four-point formulae.

$$\Delta_p = \frac{1}{4}\{B(N,Z-2) - 3B(N,Z-1) + 3B(N,Z) - B(N,Z+1)\}$$

$$\Delta_n = \frac{1}{4}\{B(Z,N-2) - 3B(Z,N-1) + 3B(Z,N) - B(Z,N+1)\}$$

The total quasiparticle (E_{qp}) energy is the sum of the one quasiparticle energy of individual nucleons. The rotational energy (E_{rot}) is given by:

$$E_{rot} = \frac{\hbar^2}{2\Im}(I(I+1) - K^2)$$

The irrotational energy (E_{irrot}) which is purely the contribution appearing from the three odd particles and is given by [5]:

$$E_{irrot} = \frac{\hbar^2}{2\Im}\left[\begin{array}{l}\left(\sum_{j_1}\left|C_{k_1}^{j_1}\right|^2 j_1(j_1+1) - k_1^2\right) \\ + \left(\sum_{j_2}\left|C_{k_2}^{j_2}\right|^2 j_2(j_2+1) - k_2^2\right) \\ + \left(\sum_{j_3}\left|C_{k_3}^{j_3}\right|^2 j_3(j_3+1) - k_3^2\right)\end{array}\right]$$

Here, $\left|C_{k_1}^{j_1}\right|^2$ are the Nilsson coefficients obtained from the Nilsson model calculations [4] by using the deformation parameters from Moller *et al.* [5]. The GM splitting and Newby shifts energies used in calculations are extracted from the available experimental bandhead energies in adjacent even-even and odd-odd nuclei [6]. The rotor-particle coupling term (E_{rpc}) gives a diagonal contribution to the bandhead energy and is given by:

$$E_{rpc} = \delta_{K,\frac{1}{2}} \frac{\hbar^2}{2\Im} (-1)^{I+\frac{1}{2}} \left(I + \frac{1}{2} \right) \times \left\{ \begin{array}{l} (\delta_{\sigma_{++-}} + \delta_{\sigma_{+-+}}) \left(\langle k_1\rho_1 | j_{1^+} | -k_1\rho_1 \rangle \delta_{k_1,\frac{1}{2}} \right) \\ +(\delta_{\sigma_{-++}} + \delta_{\sigma_{++-}}) \left(\langle k_2\rho_2 | j_{2^+} | -k_2\rho_2 \rangle \delta_{k_2,\frac{1}{2}} \right) \\ +(\delta_{\sigma_{+-+}} \delta_{\sigma_{-++}}) \left(\langle k_3\rho_3 | j_{3^+} | -k_3\rho_3 \rangle \delta_{k_3,\frac{1}{2}} \right) \end{array} \right\}$$

Results and Discussion

We updated earlier compilation of experimental data based on Three-Quasiparticle bands [7] and extracted total 228 three-quasiparticle rotational structures [8]. Out of these 228 three-quasiparticle bands, there are 76 possible K=1/2 bands and in the present paper focus only on the K=1/2 members of all the experimentally observed 3QP quadruplets. Inorder to test the validity of present version [3] of empirical model containing contributions from rotor-particle, particle-particle coupling and irrotational motion of valence protons/neutrons (all these contributions were not considered in earlier model [1]), we performed the bandhead energy calculations of K=1/2 members of $1/2[411]_\pi \otimes 7/2[523]_\pi \otimes 5/2[523]_\nu$ configuration observed in ^{175}Yb and $9/2[514]_\pi \otimes 7/2[514]_\nu \otimes 1/2[521]_\nu$ configuration observed in ^{177}Ta nuclides. The experimental data for these quadruplets is taken from ENSDF database [6]. The comparison of calculated bandhead energies with experimental data corresponding to above said configurations is given in Table 1. On the basis of results presented in Table 1, we suggest that present model calculations are in excellent agreement with experimental data as compared to earlier model calculations [1] and hence will be useful for experimentalists in predicting the K=1/2 members of 3QP quadruplets. Additionally, these calculations will also help in resolving the issues pertaining to tentative spin-parity and competing configurations assignments of K=1/2 members of a given 3QP quadruplet [8].

Acknowledgement

The financial support from Akal University, Talwandi Sabo (Bathinda), Punjab is gratefully acknowledged.

Table 3: Comparison of presently calculated bandhead energies with earlier model [1].

Nuclide	Conf.	K$^\pi$	Irrotational Correction (keV)	Residual Interaction (keV)	RPC (keV)	Bandhead Energies (keV)		
						Expt. [6]	Earlier Cal. [1]	Present Cal.
^{175}Yb	A⊗B⊗C	1/2$^+$	516.14	73.3	10.25	2114.1	1739.4	2265.8
^{177}Ta	D⊗E⊗F	1/2$^-$	430.07	12.0	30.23	1512.5	1141.9	1602.2

A: $5/2[523]_\nu$, B: $1/2[411]_\pi$, C:$7/2[523]_\pi$, D: $9/2[514]_\pi$ E: $7/2[514]_\nu$, E: $9/2[514]_\pi$, F: $1/2[521]_\nu$

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