P & D Inverse Potentials for Proton-Proton Scattering

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Introduction

A full analysis of the long-standing problem of nucleon-nucleon (NN) scattering is made possible by the different databases currently available. The VIP/GWU group [1] examined the low-energy data for $E_{Lab} \leq 450$ MeV, and the Nijmegen group [2] examined it for $E_{Lab} \leq$ 350 MeV using NN phase shift results PWA98. After that, numerous theoretical efforts were undertaken to create high-precision parameterized potentials, including the Paris potential, Nijm93, Nijm-I, Nijm-II, CD-Bonn, and many more. There are a number of free parameters in these phenomenological potentials that must be fitted to the experimental scattering data and phase shifts. Despite substantial disparities in the approaches they have used, they have virtually the same results.
Since there has never been a stable bound state of two protons, *pp* scattering experiments are the sole way to study the *pp* force. The study on the theory of pp scattering provides details on the strength and range of pp forces. The *pp* scattering analysis demonstrates that the nucleon potential is charge independent. Both *pp* and *np* scattering can use the same nuclear potential, but because cross-section depends on energy, their cross-sections are different. In addition to the nuclear interactions, the Coulomb force also contributes to the *pp* scattering.
In the recent past, Laha and group [3] used the formalisation of Supersymmetric Quantum mechanics (SQM) to generate higher partial wave potentials for NN systems and computed the scattering phase shifts using the phase function method (PFM). In this paper, we have constructed inverse potentials for P and D channels using Morse potential with Coulomb term included as an erf() function. Recent experimental data has been taken from Granada database [4]. Using PFM, our group has calculated neutron-proton [5] neutron-Deuteron[6], proton-Deuteron[7] and alpha-alpha [8] scattering phase shifts to reasonable success.

Formulation of the problem

The reference potential approach considers Morse function, given by

$$V(r) = V_0\left(e^{-2\frac{(r-r_o)}{a_o}} - 2e^{-\frac{(r-r_o)}{a_o}}\right) \quad (1)$$

as a zeroth approximation for obtaining the inverse potential corresponding to *pp* interaction. Here, the model parameters V_0, r_o and a_o represent the depth, equilibrium distance and shape of potential respectively.
The Coulomb interaction considered is of the form [5] $V_c = \frac{4e^2}{r} erf(\beta r)$ with $erf(\beta r) = \frac{2}{\sqrt{\pi}}\int_0^{\beta r} exp(-x^2)\,dx$ and $\beta = \frac{\sqrt{3}}{2R_p}$. The parameter β is inversely proportional to root mean square value of proton radius, $R_p = 0.895 fm$.

In phase function method (PFM), in order to solve for scattering phase shifts (SPS), one utilise the phase equation.

$$\delta_\ell^{'}(k,r) = -\frac{U(r)}{k}\big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr)\big]^2 \quad (2)$$

where $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Riccati-Bessel and Riccati-Neumann functions of order ℓ and the equation itself is known as Riccati-equation. Here, the initial condition

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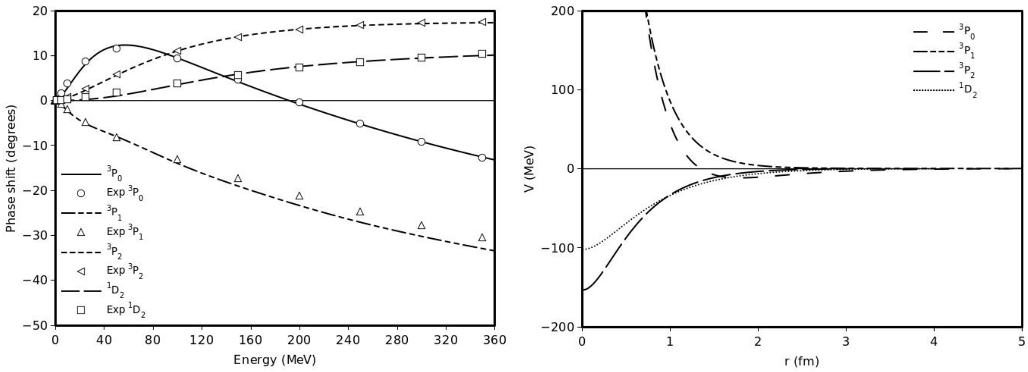

FIG. 1: Scattering phase shifts for P & D partial wave state (**left**), respective inverse potentials (**right**). The experimental scattering phase shifts are taken from Granada database [4].

is $\delta_\ell(k,0) = 0$.
By substituting $\ell = 1$ and 2 and their respective formulae for $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$, one obtain phase equations for P and D partial waves.

These phase equations for various P and D states are solved using fifth order Runge-Kutta (RK-5) method by choosing initial condition as $\delta_\ell(k, 0.01) = 0$ and varying the r value from 0.01 to 5fm in steps of 0.01. The obtained $\delta_\ell^i(k,5)$ correspond to SPS for different energies $E^i_{c.m.}$.

Results and Discussion

The model parameters for three triplet states of P and singlet state of D partial wave are optimised by minimising the mean squared error with respect to experimental data taken from Granada database [4]. The optimized parameters of P and D states are shown in Table 1. From Fig. 1 it can be clearly seen that obtained phase shifts are in good agreement with experimental phase shifts. $^1D_2$ & $^3P_2$ states have positive phase shifts and thus corresponds to negative potential. $^3P_1$ state has a negative phase shift which implies that potential must have a repulsive part which shows the correct nature of phase shift. $^3P_0$ state have positive phase shift upto 190 MeV after which SPS changes sign for $E > 190$ MeV which is an indication of pronounced repulsive core.

TABLE I: Model parameters for P & D states.

States	V_0 (MeV)	r_0 (fm)	a_0 (fm)
$^3P_0$	12.323	1.786	0.651
$^3P_1$	0.001	4.656	0.630
$^3P_2$	154.812	0.010	0.462
$^1D_2$	103.500	0.010	0.588

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