# On the $p-{}^{16}O$ elastic scattering at astrophysical energies

P. Sahoo* and U. Laha

Department of Physics, National Institute of Technology, Jamshedpur, 831014, India

*Email Id: nlphy.pati7@gmail.com

## Introduction

In the past few decades, several investigation have been made and at the same time a number of theoretical models have also been projected for the proton-nucleus scattering in the low and intermediate energy range. These surveys provide consistent data for examining the different nuclear models. In proton-nucleus scattering two potentials are involved; one is electromagnetic which takes care of the charges and the other one is nuclear in origin. The reaction $O^{16}(p,\gamma)F^{17}$ is of immense importance in stellar energy synthesis and also in the radiative capture process in nuclear physics. The form of the ${}^2S_{1/2}$ scattering phase shifts play a crucial role in the thermonuclear processes of radiative capture at astrophysical energies. Unlike the approach adapted in our previous article [1], here we take recourse to a different point of view. The electromagnetic potential is in principle well known and is the longest-range part (infinite) of the interaction. But, in reality, it is not the case. The infinite long-range part of the interaction becomes screened at some distance and thus short-range electromagnetic potentials are applied to observe the effect of screening in real situation. The major goal of the present work is to construct non-relativistic potentials with different range parameters which can be used easily in nuclear matter calculations.

We consider the potential model to map the interaction of two particle systems where nuclear part is described by short range Manning-Rosen potential and the atomic part by Hulthén potential. The short-range Manning-Rosen potential is given as

$$U_N(s)=\delta^{-2}\left[\frac{\beta(\beta-1)}{(1-e^{-s/\delta})^2}e^{-2s/\delta}-\frac{E\,e^{-s/\delta}}{1-e^{-s/\delta}}\right] \quad (1)$$

where $E$ , $\delta$ and $\beta$ are three adjustable parameters. The parameter $\delta$ has the dimension of length and $E$ & $\beta$ are dimensionless. The atomic Hulthén potential is given by

$$U_A(s)=E_0\,\frac{e^{-s/\rho}}{(1-e^{-s/\rho})} \quad . \quad (2)$$

The parameter $E_0$ defines the strength and $\rho$ stands for the screening parameter of the Hulthén potential. The s-wave Schrödinger equation for the effective potential $U_{eff}(s)=U_N(s)+U_A(s)$ , in the limit $\hbar^2/2m=1$, is written as

$$\left[\frac{d^2}{ds^2}+\xi^2-U_{eff}(s)\right]\Psi(\xi,s)=0\,. \quad (3)$$

Here the quantity $\xi$ stands for the centre of mass momentum and is related to centre of mass energy $E_{C.M.}=\hbar^2\xi^2/2m$. Introducing the following transformation

$$\Psi(\xi,s)=\delta^{\beta}\,(1-e^{-s/\delta})^{\beta}\,e^{i\xi s}\,f(\xi,s), \quad (4)$$

followed by the change of variable $(1-e^{-s/\delta})=y$ and applying the Frobenius method [2] near $y=0$ , we write

$$f(\xi,y)=\sum_{m=0}^{\infty}c_m\,y^{m+\lambda}\,,c_0\neq 0. \quad (5)$$

Solving the indicial equation, we can obtain the roots of the indicial equation and the recurrence relation. Combining the coefficients, rearranging the terms and using the basic property of Gaussian hypergeometric function, we obtain

$$\begin{aligned}\Psi(\xi,s)=&\delta^{\beta+1}(1-e^{-s/\delta})^{\beta+1}e^{i\xi s}\\ &\times\left[c_0+c_1(1-e^{-s/\delta})+c_2(1-e^{-s/\delta})^2\right. \\ &\left.+c_3(1-e^{-s/\delta})^3+c_4(1-e^{-s/\delta})^4\right]\end{aligned} \quad . (6)$$

The integral representation of the Jost function $\Im(\xi)$ for the interaction $U_{eff}(s)=U_N(s)+U_A(s)$ is written as

$$\Im(\xi)=1+\int_0^{\infty}U_{eff}(s)\,e^{i\xi s}\;\;\Psi(\xi,s)\,ds. \quad (7)$$

For our approach, the most important quantity is the Jost function $\Im(\xi)$ which can be obtained by using Eq. (6) & (7) where the roots of the Jost function in the upper complex $\xi$ -plane correspond to the bound state energies of the related system. This implies that $\Im(\xi)=0$ at $\xi=i\,k_B$.

We have applied our model to compute the elastic scattering phase shifts, excitation functions and differential cross sections of ( $p-O^{16}$ ) system at astrophysical energies, i.e., usually up to (0.6-2.0) MeV using the Jost function. In our MATLAB programme, we have used the exact value of $\hbar^2/2\mu = 22.03\ MeV\ fm^2$ for the system under consideration. For the Hulthén potential $E_0\rho = (2\xi\eta)$, where the screening radius $\rho = 55\ fm$. The quantity $2\xi\eta = 0.5255\ fm^{-1}$ and the parameters for the Manning-Rosen potential are $\beta = 0.05$, $\delta = 1.195\ fm$, $E = 0.458$. These parameters fit the correct binding energy of $F^{17}$. Results of our phase analysis for the $p - O^{16}$ system are presented in Table-1 and compared with experimental results of Dobovichenko *et al.* [3].

**Table 1.** The ( $p - O^{16}$ ) scattering phase shifts along with the Ref. [3].

| $E_{Lab}$ (MeV) | Phase shift $\delta^+_{1/2}$ (degree) Present work | Phase shift $\delta^+_{1/2}$ (degree) Ref. [3] | $E_{Lab}$ (MeV) | Phase shift $\delta^+_{1/2}$ (degree) Present work | Phase shift $\delta^+_{1/2}$ (degree) Ref. [3] |
|---|---|---|---|---|---|
| 0.385 | 182.92 | 179.59 | 0.905 | 168.05 | 174.19 |
| 0.487 | 179.57 | 179.74 | 0.979 | 166.48 | 172.17 |
| 0.616 | 176.09 | 179.65 | 1.106 | 164.21 | 169.82 |
| 0.663 | 174.72 | 178.11 | 1.250 | 161.94 | 166.43 |
| 0.716 | 172.30 | 177.96 | 1.370 | 160.19 | 163.61 |
| 0.759 | 171.24 | 176.46 | 1.589 | 158.48 | 159.42 |
| 0.810 | 169.87 | 174.55 | 1.790 | 155.14 | 155.78 |
| 0.861 | 168.82 | 173.37 | 1.990 | 152.05 | 151.09 |

Using the phase parameters as given in Table 1 for $p - O^{16}$ system we have computed elastic differential cross section and excitation functions and the obtained results are in good agreement with Dobovichenko *et al.* [3] & Chow *et al.* [4] which are portrayed in Figures 1-2. The results of phase shifts of $(p\text{–}^{16}O)$ elastic scattering will allow in the future to parameterize complex interaction potentials for scattering processes in a non-resonant $^2S_{1/2}$ state. These potentials, in turn, may further be used in calculations of various astrophysical problems, such as radiative capture of particles on light nuclei, and in this case, proton capture on $O^{16}$. The Jost function approach is more convenient and straightforward for studying both bound and scattering state problems. The problem of charged particle interaction with this proposed potential model having different range parameters works satisfactorily.

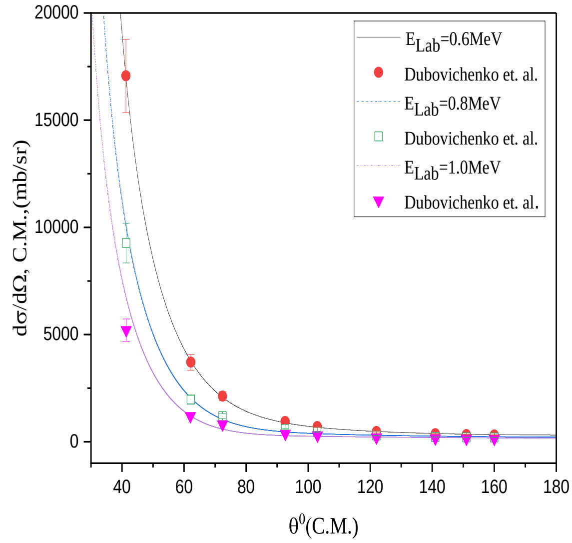


**Fig.1.** Angular distributions of $p\text{–}^{16}O$ elastic scattering at $E_{Lab} = 0.60, 0.80\ \&\ 1.0 MeV$ along with experimental results of Dobovichenko *et al.* [3].

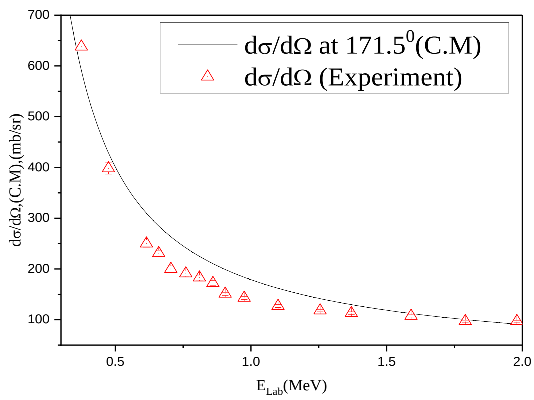


**Fig.2.** Excitation functions at $171.5^0$ (C.M.) along with the Ref. [4].

## References


**[1]** P. Sahoo and U. Laha, *Pramana-J. Phys.* **96**, 15 (2022)

**[2]**G. B. Arfken, *Mathematical Methods for Physicists*, Academic Press, New York (1985)

**[3]** S. Dubovichenko et al., *Chinese Phys. C* **41**, 014001 (2017)

**[4]** H. C. Chow et al., *Can. J. Phys.* **53**, 1672 (1975)