Coulomb-mediated entanglement between two nuclei in free fall

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The quantum entanglement of two objects is defined as a type of correlation where the act of measurement on a subsystem immediately affects the state of the other, even if they are separated by a distance. While most quantum technologies are based on discrete spin states, entanglement in continuous variables is regarded as the holy grail of quantum information theory.

It has been demonstrated in Ref. [1] that the Coulomb interaction between mesoscopic particles accumulates a significant amount of entanglement, even when they are confined in harmonic traps. However, the entanglement gain is amplified by letting go of the harmonic confinement [2], at least along the line joining their centers. In this work, we implement the ideas of Ref. [3] in a hypothetical experiment of two identical nuclei released from the ground state of harmonic traps. Once prepared, the traps are switched off, and the particles interact with each other in free fall. Since the harmonic oscillator ground state is a Gaussian, the bipartite state at $t = 0$ is a two-mode Gaussian.

Formalism

Following the seminal approach as in Refs. [2, 3] we write the Hamiltonian in the displacement space

$$\hat{H} = \frac{\hat{p}_A^2}{2m} + \frac{\hat{p}_B^2}{2m} + Z_A Z_B \frac{\alpha\hbar c}{L + (\hat{x}_B - \hat{x}_A)}, \quad (1)$$

where L is the separation between the two nuclei, $\hat{x}_A$ and $\hat{x}_B$ are their displacements from initial mean positions, with $\hat{p}_A$ and $\hat{p}_B$ their respective momenta. Z_A and Z_B are their atomic numbers, and $\alpha \approx 1/137$ is the fine-structure constant that characterizes the strength of Coulomb interaction. The two-mode Gaussian is now transformed from the LAB to the COM frame of reference. Unlike the regular literature on scattering theory, the COM can no longer be described as a plane wave [3]. In fact, at $t = 0$, it is a Gaussian wave packet of width, just like the reduced mass. The Hamiltonian in this frame decouples as

$$\hat{H} = \left(\frac{\hat{P}^2}{4m}\right) + \left(\frac{\hat{p}^2}{m} + Z_A Z_B \frac{\alpha\hbar c}{L + \hat{r}}\right). \quad (2)$$

where r is the displacement of the reduced mass from its initial average location. Accordingly, the COM behaves as a Gaussian wave packet expanding in the free space, and reduced mass evolves in time under the influence of the Coulomb potential, which can be further expanded in a Taylor series:

$$\hat{H}_r = \frac{\hat{p}^2}{m} + Z_A Z_B \frac{\alpha\hbar c}{L} \sum_{n=0}^{N} (-1)^n \frac{\hat{r}^n}{L^n}. \quad (3)$$

where N is the order of interaction. Given that the potential is truncated at the second order, the time-evolution conserves the Gaussianity of the bipartite quantum state. Accordingly, the state of the system at any given time is fully described by the first two statistical moments (variances and correlations). In other words, the covariance matrix encodes all relevant information as there is in the density matrix. For a bipartite system, the covariance matrix is given by

$$\boldsymbol{\sigma_{ij}} = \frac{1}{2} \langle \hat{u}_i \hat{u}_j + \hat{u}_j \hat{u}_i \rangle - \langle \hat{u}_i \rangle \langle \hat{u}_j \rangle, \quad (4)$$

where $\hat{u} = (\hat{x}_A, \hat{p}_A, \hat{x}_B, \hat{p}_B)$.

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Results and discussion

We assume the potential to be truncated up to the second order and employ the techniques of Ref. [3] to derive a time-dependent analytical solution for the covariance matrix. The same is then utilized to quantify the entanglement with logarithmic negativity [4] and von-Neumann entropy of entanglement [5]. For a demonstration, we consider two ^{196}Au nuclei, nuclei separated by a distance of 2 μm and released after cooling in harmonic traps of frequency 10^{12} Hz. The logarithmic negativity and the entanglement entropy are shown in Fig. 1. It can be easily seen that the Coulomb interaction creates a huge amount of entanglement, that too within a nanosecond!

It is proved in Ref. [3] that force gradient is the dominant contributor to position-momentum correlations, and the inclusion of higher than quadratic term in the expansion s are crucial for obtaining the momentum dependence of the entanglement gain. An extension of our methods in this way will reveal the entanglement gain in the Coulomb scattering. It is worth pointing out that nuclear experiments are performed at room temperature only, and the effects of decoherence must be incorporated. Also, while the positions and momenta of mesoscopic objects can be measured with weak probing lasers, some alternatives must be implemented for the nuclei.

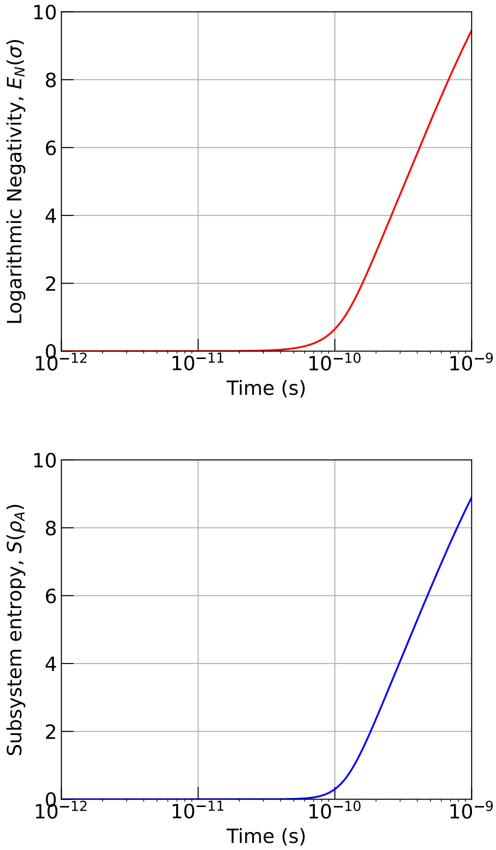

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FIG. 1: Coulomb-mediated entanglement between two Gold nuclei released after cooling in harmonic traps. The separation between the nuclei is 2 μm, and the trapping frequency is 10^{12} Hz. Coulomb potential is truncated up to the second order in the displacement, and the entanglement quantifiers are calculated within the covariance matrix formalism.

Conclusions

We calculated the Coulomb-mediated gain of Gaussian entanglement between two nuclei. The Coulomb interaction is treated as a direct coupling in the bipartite system of two Gold nuclei released after cooling in harmonic traps. It is shown that the Coulomb interaction generates a huge amount of entanglement within a nanosecond. The impacts of decoherence due to temperature and an extension to incorporate momentum dependence will reveal the gain of entanglement in realistic Coulomb scattering experiments.

References

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