Charge radii of Δ^{++} in statistical model

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I. INTRODUCTION

Understanding the structure of baryons is still a daunting task in quantum chromodynamics (QCD). It can be determined in terms of electromagnetic Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ [1]. The electromagnetic form factors are related to the static low-energy observables. Many theoretical formalisms have investigated these observables such as FTQM [2], chiral perturbation theory [3], lattice QCD [4], $1/N_c$ expansion [5] etc. The statistical model has already been successful in calculating and analyzing the properties in low energy regime [6]. In view of above developments in the statistical model, it therefore becomes desirable to extend it to calculate the charge radii of decuplet baryons. The mean square charge radii (r_B^2) is the lowest order moments of the charge density in a low-momentum expansion. It contains fundamental information about the possible "size" of the baryons. The mean square radius of a given baryon is defined as:

$$\langle r^2 \rangle = \int d^3 r \rho(r) r^2 \quad (1)$$

where $\rho(r)$ is the charge density.
Morpurgo et. al [7] developed a general parameterization method (GPM) to calculate different properties like quadrupole moments, masses, charge radii etc. Here, we have extended this work in the statistical approach to calculate the charge radii of Δ^{++}. The general form of charge radii operator composed of the sum of one, two, and three-quark systems can be defined as:

$$r_B{}^2 = A \sum_i e_i.1 + B \sum_{i \neq j} e_i \sigma_i \sigma_j + C \sum_{i \neq j \neq k} e_i \sigma_j \sigma_k \quad (2)$$

Here A, B, and C are the parameters that can be calculated from the recent experimental data on charge radii. The above expression can be written in simplified [8] form as:

$$r_B{}^2 = (A - 3B + 6C) \sum_i e_i + 5(B - C) \sum_i e_i \sigma_{iz} \quad (3)$$

The z-component of the Pauli spin (isospin) matrix σ_i is denoted by σ_{iz} and e_i is the charge of the i-th quark where i = (u,d,s) for any of the three quarks.

II. THEORETICAL FRAMEWORK

With the extension of Naive Quark Model (NQM), the hadronic structure formed with valence quarks coupled with "sea quarks". The statistical model is based on the assumption that hadrons as an ensemble of quark-gluon Fock states. Each Fock state contributes to some part of the total probability associated with quark-gluon Fock states. The methodology is based on applying the charge radii operator defined in eqn. (3) to the wave function of decuplet baryons. A suitable wave-function is framed with the inclusion of sea containing quark-gluon Fock states. The sea considered here is consist of quark-antiquark pairs muticonnected through gluons. The wave-function contains suitable combinations of valence and sea quarks to maintain the overall anti-symmetrization (spin 3/2, flavor decuplet, color singlet) of the baryonic

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system and is represented as:

$$|\Phi_{3/2}^{(\uparrow)}\rangle = \frac{1}{N}[a_0\Phi_1{}^{(\frac{3}{2}\uparrow}H_0G_1 + b_1(\Phi_8{}^{(\frac{1}{2})}\otimes H_1)^{\uparrow}G_1 + b_8(\Phi_8{}^{(\frac{1}{2})}\otimes H_1)^{\uparrow}G_8 + d_1(\Phi_1{}^{(\frac{3}{2})}\otimes H_2)^{\uparrow} G_1 + d_8(\Phi_8{}^{(\frac{1}{2})}\otimes H_2)^{\uparrow}G_8] \quad (4)$$

Where $N^2 = a_0^2 + b_1^2 + b_8^2 + d_1^2 + d_8^2$
N is the normalization constant and a_0, b_1, b_8, d_1, d_8 are the coefficients that are associated with each term in the wave-function. They define the probability associated with relevant Fock states. The details explanation of the above wavefunction can be seen in ref. [9]. The charge radii r_B^2 can now be calculated by operating the charge radii operator on every term of the above-mentioned wavefunction and is given as:
$\boldsymbol{r_B^2 = \langle\Phi_{3/2}^{(\uparrow\ )}|\hat{O}|\Phi_{3/2}^{(\uparrow\ )}\rangle}$,
where $\hat{O}$ denotes the charge radii operator.

III. RESULT AND CONCLUSION

The results for charge radii of Δ^{++}, in the statistical model discussed in this section. On applying the given operator on the wavefunction. The contribution comes in the sets of two parameters, one corresponds to the statistical parameters $(a_0, b_1, b_8...)$ and the other one is GPM parameters (A, B, and C).

(2A+4B+2C)a_0 + (0.33A+6.34B- 5.33C)b_1 +(0.42A +6.74B - 5.5C)b_8....

For statistical parameters, we used the same values as discussed in ref. [10] and at the end, we get the expression :

$$1.62A + 5.29B - 0.82C \quad (5)$$

The table given below shows the calculated values of charge radii in terms of GPM parameters and the comparison with other models. We performed fitting to the available experimental observations on charge radii and get the values of the parameters for Δ^{++} is:
A = 1.612, B = -0.305, C = -0.065

Charge radii (r_B^2)	Statistical model	FTQM	χCQM	$1/N_c$
Δ^{++}	1.048	1.18	0.938	1.011

We can further extend this calculation to obtain the charge radii for other decuplet members. To summarize, the statistical model is able to provide a suitable match of charge radii for Δ^{++} including the effect of the sea.

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