Spectroscopy of B_c meson in an instanton plus confinement potential model

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Introduction

The B_c meson family aroused special scientific interest among the heavy flavour systems as they carry different flavour quarks which provide new opportunities to test heavy quark dynamics in QCD. Also, the latest observations of CMS and LHCb has renewed intrest in the study of B_c mesons [1]. In the present study we invoke the effect of instanton potential along with confinement for the realistic estimation of B_c nS states inspired from our recent work for bottomonium [2].

Theoretical Formalism

QCD is the non-abelian gauge theory pertaining many of the exciting topological properties. One of such properties is the nontrivial solution of the Yang-Mills field equations in the Euclidean space. These solutions are called the instantons. The instanton liquid model of QCD vacuum (ILM) has two important parameters: the average size ($\bar{\rho}$) and the average distance between the instantons $(\bar{R})$. The second parameter $\bar{R}$ also represents the density of the instanton media as $(N/V)^{1/4} \equiv \bar{R}$; where N is the number of instantons and V is the volume of instanton ensemble. Different values of these parameters are found in literature listed in Table I. It would be very interesting to see the effect of different set of parameters on the predictions of B_c states particularly the $\bar{R}$. The central part of the quark anti-quark interaction from the instanton liquid model is given by [2, 3]

$$V(r) = \frac{4\,\pi\bar{\rho}^3}{\bar{R}^4 N_c} I\left(\frac{r}{\bar{\rho}}\right) \tag{1}$$

For $r \ll \bar{\rho}$ i.e., when the distance between quark-antiquark is smaller than the average size of instanton, the central potential is given by [2, 3]

$$V(r) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^3} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \tag{2}$$

here, $N_c = 3$ represents the colour degrees of freedom. For $r \gg \bar{\rho}$ i.e., the distance between quark-antiquark is higher than the size of instanton, the central part of the potential is given by [2, 3]

$$V(r) \simeq 2\Delta M_Q - \frac{g_{np}}{r} \tag{3}$$

Within ILM framework potential range is defined only for $r \ll \bar{\rho}$ and $r \gg \bar{\rho}$ which leads to kink at $r = \bar{\rho}$. To remove that discontinuity we need to add V_1 in Eq. 3. The values of V_1 for M-I, M-II and M-III are also listed in Table I. As $r \to \infty$, potential is saturated at $2\,\Delta M_Q$ means that it can not explain the confinement [2]. Hence, for the present study, we have added a state dependent confinement potential V_0 which has the form

$$\begin{aligned} V_0(z) &= b\ ln\,(z) + a \\ &= b\ ln\,(2\ N + l - 1) + a \end{aligned} \tag{4}$$

Where, $z = (2N + 3) - (4 - l)$ and $N = 2n + l$. The Schrödinger equation is solved based on variational method using instanton potential and 3D harmonic oscillator trial wave function basis set [2].

Results and Discussion

In the present study we have computed the masses of bottom-charm mesons within the instanton liquid model. For the M-I parameters,

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TABLE I: The input parameters used in present study with $m_b = 4.18$ GeV and $m_c = 1.27$ GeV

	M-I	M-II	M-III
$\bar{\rho}$ (in fm)	0.35 [6]	0.36 [2, 7]	0.33 [5]
$\bar{R}$ (in fm)	0.85 [6]	0.89 [2, 7]	1.00 [5]
V_1 (in MeV)	89.74	83.74	40.47
b (in MeV)	368.16	391.07	477.11
a (in MeV)	-47.45	-60.25	-131.49

TABLE II: $1S$ and $2S$ mass (in MeV) of B_c states from Instanton potential Model without confinement and Instanton Model with confinement presented as C

	1^3S_1	1^1S_0	2^3S_1	2^1S_0
M-I	5741	5733	5898	5891
M-II	5723	5716	5869	5862
M-III	5629	5625	5691	5688
M-I+C	6312	6274	6908	6881
M-II+C	6312	6274	6911	6884
M-III+C	6312	6274	6975	6946
Exp [4]	…	6274.9 ± 0.8	…	6871 ± 1.1
[9]	6326	6271	6890	6871
[10]	6328	6277	6898	3868

one can find the kink of 89.93 MeV. While for M-II and M-III these values are 83.74 MeV and 40.47 MeV. The value of V_1 is smaller as the value of $\bar{R}$ increases (less dilute medium of instanton). One can see from Table II that predicted masses without confinement are significantly low as compared to the experimental values [4]. It is seen from Figure 1 that as $\bar{R}$ increases strength of potential decreases. Hence, the small value of $\bar{R}$ is essential for hadron spectroscopy. We have added additional confinement potential in order to get the realistic B_c mass as presented in Table II. The B_c^* state has not been observed yet so we fix the known B_c state which gives the value of centroid ($M(1S)$) as 6301 MeV for all the models. This corresponds to the mass of state $B_c^* = 6310$ MeV for M-I, M-II and M-III. This mass is also falls in the range quoted by Kwong and Rosner given by 6284 MeV $\leq M(B_c^*) \leq$ 6357 MeV [8]. The hyperfine splitting $M(B_c^*) - M(B_c)$ found to be 38 MeV. We also see that additional amount of confinement is required more for increased value of $\bar{R}$. The predicted value of 2^1S_0 from M-I and M-II are 6881 MeV and 6884 MeV respectively are close to PDG listed average value of 6871 ± 1.1 MeV [4] whereas M-III predictions found to be higher. The predicted mass of 2^3S_1 from both M-I and M-II are found to be close to results of non-relativistic quark model [9] and constituent quark model [10]. In summary, we emphasize that less dilute medium of instanton is required for the hadron spectroscopy. Also, the value of $\bar{R}$ used in formalism affects the fine-tuning of the mass spectra significantly.

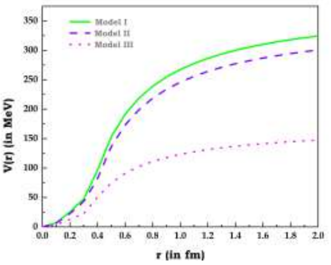

FIG. 1: The strength of the instanton induced interaction potential as a function of inter quark distance form M-I to M-III

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