Solution of the Harmonic plus Modified Yukawa-Kratzer Potential and its application in hadron Physics

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Introduction

The concept of hadrons was introduced in 1962 by Okun [1] and developed into the quark model in 1964 independently by Gell-Mann [2] and Zweig [3], describing ordinary mesons ($q\bar{q}$) and baryons (qqq) in terms of quarks q and antiquarks $\bar{q}$. In addition to the quark model, the possible existence of exotic hadrons, such as tetraquarks ($q\bar{q}q\bar{q}$) or ($qq\bar{q}\bar{q}$) and pentaquarks ($q\bar{q}qqq$), Mann's seminal work [2], but it was not until the beginning of the 21st century when the first claimed observations of exotic hadrons were made [4]. Concerning tetraquarks, the first discovery was made in 2003 by the Belle Collaboration which observed a resonance peak at (3872±0.6) MeV [5], which was named X(3872). Rai et al.[6] studied Tetraquark, Pentaquark, and Hexaquark systems are investigated as di-hadronic molecules in a non-relativistic model and low-lying di-hadronic state in a relativistic harmonic model [7] We investigate a nonrelativistic model of tetraquarks, which are assumed to be compact and to consist of diquark-antidiquark pairs, and study the ground-state mass of full-heavy tetraquarks $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ with solving the nonrelativistic four-body systems. New analytic exact energy eigenvalues and eigenfunctions are obtained in fractional forms using the Schrodinger equation via the extended Series expansion method. The Schrodinger equation or Schrodinger-type equation [8–11] can be solved analytically or numerically to offer us a lot of information about a physical system because of their importance in Nuclear and particle physics, quantum field theory, statistical physics, etc... We have used heavy tetraquark systems to verify the method's applicability. The present results show a good agreement with other studies. We conclude that the models play a good role in heavy tetraquark masses.

Solution of the HMYKP using series expansion method

We consider the radial Schrodinger equation [12]

$$\frac{d^2u_{nm}(r)}{dr^2}+\frac{2}{r}\frac{du_{nm}(r)}{dr}+$$

$$\left[-E_{nl}^{'}-V_{eff}(r)\right]u_{nm}(r)=0 \qquad (1)$$

The Harmonic plus Modified Yukawa-Kratzer potential (HMYKP) can be written as,

$$V(r)=a_1r^2+\frac{a_2e^{-2\alpha r}}{r^2}-\frac{a_3e^{-\alpha r}}{r}$$

$$+a+D_e-\left(\frac{A_1}{r}-\frac{A_2}{r^2}\right) \qquad (2)$$

Where, $A_1=2D_er_e$, and $A_2=D_er_e^2$

$$V_{eff}=(a_1+C_1)r^2+C_2r-$$

$$\frac{C_3}{r}+\frac{C_4}{r^2}+C_5 \qquad (3)$$

Where

$$C_1=\left[\frac{2}{3}\beta_1+\frac{\beta_2}{24}\right]\alpha^2, C_2=\left[-\frac{4}{3}\beta_1-\frac{\beta_2}{6}\right]\alpha$$

$$C_3=\frac{2\beta_1+\beta_2}{\alpha}, C_4=\frac{\beta_1+\beta_2+\beta_3}{\alpha^2} \qquad (4)$$

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The mass spectra of heavy tetraquark systems such as $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ which have the same flavor quark and antiquark, are obtained from the mass spectra.

$$M = 2m + a + D_e +$$

$$\frac{\hbar^2}{2\mu}\sqrt{\left[\frac{2}{3}\beta_1 + \frac{\beta_2}{24}\right]\alpha^2\left(4n + 2 + 2\Omega\right)} -$$

$$\frac{\hbar^2}{2\mu}\frac{\left(2\beta_1 + \beta_2\right)^2}{4\alpha^2}\left(4n + 1 + 2\Omega\right)^{-2}$$

$$+\frac{\hbar^2}{2\mu}\left(2\beta_1 + \frac{\beta_2}{2}\right) \quad (5)$$

Discussion of Results

TABLE I: The mass spectra of $cc\bar{c}\bar{c}$ (in GeV) for parameter $M_{cc\bar{c}\bar{c}}$ =6.905, $\hbar$, μ, a = 1, D_e=1.5, α=0.1 and comparison with other works.

State	Our	[13]	[14]	[15]
1S	6.123	6.198	5.966	6.332
1P	6.437	-	6.462	-
2S	6.658	-	6.669	6.575
2P	6.852	-	6.830	-
3S	6.981	-	7.011	6.782
4S	7.233	-	-	-

We propose to develop a diquark-antidiquark system as a four-particle, two-body system to investigate the tetraquark system. The idea of tetraquarks as diquark-antidiquark systems is used to construct a technique for dictating the quantitative masses of specific tetraquark states. The corresponding energy eigenvalues were obtained, which depend on the fractional parameters α and β. The correlation of our results with other conventional models revealed an acceptable level of agreement. This leads to the study of the analytical solution of the fractional Schrodinger radial equation at present potential under generalized fractions can provide valuable information on particle physics.

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