Instanton Contribution to the Bc Meson Mass Spectra*

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Introduction

The Bc states are composed of a bottom-charm quark-antiquark pair, provides an scope for study of non-relativistic interactions among mesonic systems. It was predicted as an important family of hadron spectra in theory about more than 40 years ago [5]. The CDF Collaboration at Fermilab observed the ground state of Bc meson in 1998 [2]. In 2018, the LHCb Collaboration observed the existence of excited B state with a mass of 6842±9 MeV by using their 8 TeV data sample, until then, only the ATLAS Collaboration reported evidence of an excited state [3]. The difficulties in observations and measurements of the Bc spectrum is due to the production yields being significantly smaller than those of the charmonium and bottomonium states.

Theoretical Background

In a potential model approach the entire dynamics of quarks in a meson is governed by a Hamiltonian has kinetic energy term (K) and a potential energy (V). The potential energy V is the sum of the heavy-quark potential $V_{Q\bar{Q}}(\vec{r})$, confining potential $V_{conf}(\vec{r})$ and Coulomb potential $V_{coul}(\vec{r})$.

$$\begin{aligned} V(\vec{r}) &= V_{Q\bar{Q}}(\vec{r}) + V_{coul}(\vec{r}) + V_{conf}(\vec{r}) \\ V_{Q\bar{Q}}(\vec{r}) &= V_C(\vec{r}) + V_{SD}(\vec{r}). \end{aligned}$$

Here $V_C(\vec{r})$ and $V_{SD}(\vec{r})$ are central and spin dependent potentials due to instanton vacuum respectively [9].

$V_C(\vec{r})$ is given by the following expression

$$V_C(\vec{r}) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^2} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \quad (1)$$

Here, $\bar{\rho} = \frac{1}{3}$ fm the average size of the instanton, $\bar{R} = 1$ fm the average separation between instantons and number of colors N_C is 3.

The spin-spin interaction $V_{SS}(\vec{r})$, the spin-orbit coupling term $V_{LS}(\vec{r})$ and the tensor part $V_T(\vec{r})$ contribute to the spin dependent potential $V_{SD}(\vec{r})$;

$$\begin{aligned} V_{SD}(\vec{r}) &= V_{SS}(\vec{r}) + V_{LS}(\vec{r}) + V_T(\vec{r}) \\ V_{SS}(\vec{r}) &= \frac{1}{3{m_Q}^2}\nabla^2 V_C(\vec{r});\ V_{LS}(\vec{r}) = \frac{1}{2{m_Q}^2}\frac{1}{r}\frac{dV_C(\vec{r})}{dr}; \\ V_T(\vec{r}) &= \frac{1}{3{m_Q}^2}\left(\frac{1}{r}\frac{dV_C(\vec{r})}{dr} - \frac{d^2V_C(\vec{r})}{dr^2}\right). \end{aligned}$$

The coulomb-like (perturbative) one gluon exchange part of the potential is given by

$$V_{coul}(\vec{r}) = \frac{-4\alpha_s}{3r} \quad (2)$$

with the strong coupling constant α_s and inter quark distance r. The confinement term represents the non perturbative effect of QCD which includes the spin-independent linear confinement term [39].

$$V_{conf}(\vec{r}) = -\left[\frac{3}{4}V_0 + \frac{3}{4}cr\right]F_1 \cdot F_2 \quad (3)$$

where c and V_0 are constants. F is related to the Gell-Mann matrix, $F_1 = \frac{\lambda_1}{2}$ and $F_2 = \frac{\lambda_2^*}{2}$ and $F_1 \cdot F_2 = \frac{-4}{3}$ for the mesons.

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Results and Discussions

In this model of nonrelativistic dynamics, instanton induced potential and Coulomb like OGEP plays superior role. The instanton effects on masses of orbitally excited state of Bc mesons are listed in the table 2.

Table 1 The contributions from instantons (M_I) (MeV)

$n^{2S+1}L_J$	The Mass	M_{exp} MeV	M_I MeV
$1\,{}^1S_0$	6268	6274.47±0.32	11.28
$2\,{}^1S_0$	6856	6871.2±1.0	14.69
$3\,{}^1S_0$	7212	-	17.60
$4\,{}^1S_0$	7483	-	18.24
$5\,{}^1S_0$	7920	-	17.48
$1\,{}^3S_1$	6349	-	10.14
$2\,{}^3S_1$	6895	-	14.52
$3\,{}^3S_1$	7261	-	17.46
$4\,{}^3S_1$	7683	-	18.32
$5\,{}^3S_1$	7880	-	17.18

Table 2 The contributions to the excited states (MeV)

$n^{2S+1}L_J$	The Mass	M_{exp} (MeV)	M_I (MeV)
$1\,{}^3P_0$	6705	-	5.92
$2\,{}^3P_0$	7099	-	7.89
$3\,{}^3P_0$	7473	-	8.99
$1P$	6725	-	6.28
$2P$	7135	-	8.14
$3P$	7525	-	9.58
$1P'$	6748	-	6.01
$2P'$	7167	-	8.02
$3P'$	7543	-	9.05
$1\,{}^3P_2$	6717	-	6.13
$2\,{}^3P_2$	7157	-	8.31
$3\,{}^3P_2$	7456	-	9.15
$1\,{}^3D_1$	6952	-	3.42
$2\,{}^3D_1$	7371	-	7.03
$3\,{}^3D_1$	7775	-	10.45
$1D$	6952	-	3.14
$2D$	7370	-	6.86
$3D$	7783	-	10.32
$1D'$	7063	-	3.22
$2D'$	4707	-	6.91
$3D'$	7788	-	10.42
$1\,{}^3D_3$	7022	-	3.28
$2\,{}^3D_3$	7402	-	7.10
$3\,{}^3D_3$	7791	-	11.39

The spin dependent terms of instanton potential give the hyperfine splitting. The OGEP is required as it is consistent with the asymptotic freedom.

Instantons were introduced in relation to the UA(1) problem and their role was pointed out by t'Hooft by deriving effective interactions by coupling of the instantons and light quarks, whose strength of interaction depends on the instanton density, which was estimated from the gluon condensate of the QCD vacuum. It was argued that the NRQM should include the instanton potential as a short-range non-perturbative gluon effect.

The discrepancies in the theoretical predictions may vary from one model to the other due to the choice of potential which plays an important role in the predictions of mass which in turn creates differences in phase space which affects the transition widths. Also sometimes it is a choice of different wave function which effects the predictions of the transition widths. Also, instanton vacuum ptential plays a vital role in obtaining mass spectroscopy and other relevent properties of bottomonium. Finally, we hope that our predicted results using instanton effects on heavy quarks will be helpful in search of the new quarkonium physics experimentally as well as theoretically.

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