On far-from-equilibrium applicability of hydrodynamics in heavy-ion collisions

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The success of relativistic fluid dynamics in describing the matter formed in ultra-relativistic heavy-ion collisions initially led to the belief that the hot and dense matter formed in these collisions is close to local thermal equilibrium [1]. However, with the advent of numerical dissipative hydrodynamic models, it became clear that the evolution of the formed matter is affected by large dissipative corrections. This "unreasonable effectiveness" of hydrodynamics in providing a dynamical description of high-energy collisions has generated much interest in the formulation of new fluid dynamic theories [2]. In the present study, we explore the approach to the hydrodynamic regime of a plasma of massive particles that mimics the matter created in the collision of heavy nuclei at high energy [3].

We consider a fluid consisting of particles of mass m undergoing a boost-invariant expansion along the z direction [4]. The kinetic equation describing the evolution of the distribution function, in the relaxation-time approximation is [5]

$$\left(\frac{\partial}{\partial\tau}-\frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right)f(\tau,p)=-\frac{f(\tau,p)-f_{\rm eq}(p_0/T)}{\tau_R}, \quad (1)$$

where τ is the proper time, τ_R denotes the relaxation time, and $p_0 = \sqrt{m^2+p^2}$ is the energy of a particle with momentum p. The energy-momentum tensor can be obtained from the single-particle distribution function: $T^{\mu\nu} = \int dP\, p^\mu p^\nu f(\tau,p)$, where $dP = d^3p/[(2\pi)^3 p_0]$.

For a system undergoing Bjorken expansion, the components of energy-momentum tensor (ϵ, P_T and P_L) can be defined in terms of the moments

$$\mathcal{L}_n \equiv \int dP\, p_0^2\, P_{2n}(\cos\psi)\; f(\tau,p)\,, \quad (2)$$

where P_{2n} is the Legendre polynomial and $\cos\psi \equiv p_z/p_0$. The first two $\mathcal{L}_n$-moments are $\mathcal{L}_0 = \epsilon$ and $\mathcal{L}_1 = (3P_L-\epsilon)/2$. The transverse pressure involves in addition the trace of the energy-momentum tensor, $P_T = \frac{1}{3}\left(\mathcal{L}_0-\mathcal{L}_1-\frac{3}{2}T^\mu_\mu\right)$, which cannot be expressed solely in terms of the $\mathcal{L}_n$-moments. This requires another type of moments, which we define as

$$\mathcal{M}_n \equiv m^2 \int dP\, P_{2n}(\cos\psi)\; f(\tau,p). \quad (3)$$

The moment $\mathcal{M}_0$ is equal to the trace of the energy-momentum tensor T^μ_μ. The bulk viscous pressure (Π), and a single independent shear stress tensor component (ϕ) can be expressed in terms of the moments and the equilibrium pressure (P),

$$P+\Pi=\frac{1}{3}(\mathcal{L}_0-\mathcal{M}_0),\ \phi=-\frac{2}{3}\left(\mathcal{L}_1+\frac{\mathcal{M}_0}{2}\right). \quad (4)$$

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Using the relations among the Legendre polynomials and the definitions (2), (3) of the moments, the kinetic equation (1) can be recast into a hierarchy of coupled equations:

$$\frac{\partial \mathcal{L}_0}{\partial \tau} = -\frac{1}{\tau}\left(a_0\mathcal{L}_0 + c_0\mathcal{L}_1\right), \tag{5a}$$

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left(a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right) - \frac{(\mathcal{L}_n - \mathcal{L}_n^{\rm eq})}{\tau_R}, \tag{5b}$$

$$\frac{\partial \mathcal{M}_n}{\partial \tau} = -\frac{1}{\tau}\left(a'_n\mathcal{M}_n + b'_n\mathcal{M}_{n-1} + c'_n\mathcal{M}_{n+1}\right) - \frac{(\mathcal{M}_n - \mathcal{M}_n^{\rm eq})}{\tau_R}, \tag{5c}$$

where the coefficients a_n, b_n, c_n and a'_n, b'_n, c'_n are real constants. The equations for the lowest three moments $\mathcal{L}_0$, $\mathcal{L}_1$, and $\mathcal{M}_0$ fully represent the evolution of energy-momentum tensor. Israel-Stewart-like (ISL) second-order non-conformal hydrodynamic equations obtained in [6] can be derived from these moment equations. The derivation is based on the moment truncation, in particular, accounting for the moment $\mathcal{L}_2$ and $\mathcal{M}_1$ [3].

We compare results obtained by solving the second-order hydrodynamic equations [6] with those obtained from the exact solution of the kinetic equation in Fig. 1 for two physical quantities, the longitudinal (P_L) and the transverse (P_T) pressures. The initial condition is set at $\tau_{\rm in} = 0.1\tau_R$, with an isotropic distribution function. The red and green lines correspond to $z \equiv m/T = 1$ and 0.01, respectively, at time $\tau = \tau_R$. Remarkably, as can be seen in the figure, the exact results are also approximately reproduced by second-order ISL hydrodynamics. In particular, a short free-streaming regime (dotted curves) is seen in both the kinetic description and in the ISL hydrodynamic one. There is of course nothing typically "hydrodynamic" here; hydrodynamics becomes a valid description only for times $\tau \gtrsim \tau_R$. The reason ISL theories capture such free-streaming behavior, albeit approximately, is because the relaxation-type structure of ISL equations is similar to that of the truncated three-moment equations. The time derivative of the moments $\mathcal{L}_1$ and $\mathcal{M}_0$, and correspondingly, the viscous pressures, capture approximately some of the features of the collisionless regime of the expanding system [3].

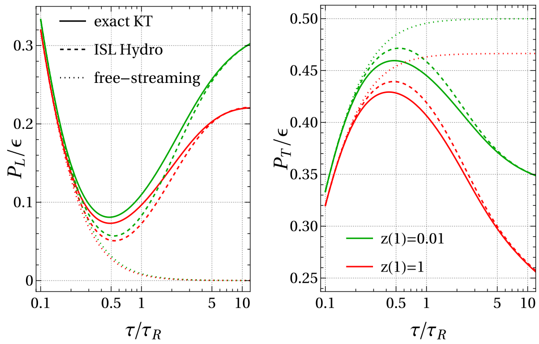

FIG. 1: The longitudinal (P_L) and the transverse (P_T) pressures normalized to the energy density ϵ, as a function of time [3].

The work reported here sheds light on the "unreasonable success of hydrodynamics" in describing the space-time evolution of the nuclear matter formed in heavy-ion collisions even in far-from-equilibrium situations. This success may be attributed to the phenomenological Israel-Stewart-like theories which are used in simulations of heavy-ion collisions, and not to hydrodynamics.

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