Study of ring–like and jet–like events in heavy–ion collisions using R/S analysis technique

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Introduction

The intricate dynamics of multiparticle production processes can be studied using heavy-ion collisions at relativistic energies. Two types of substructures were revealed in relativistic heavy ion collisions, which might be referred to as "jet-like" and "ring-like" substructures [1]. The "ring-like" substructures are the result of multiple pions being produced in confined places along the rapidity (y) axis and then diluted across the entire azimuth. In the case of "jet-like" substructures, pions are emitted in a narrow area along both pseudorapidity (η) and azimuth (ϕ).

In the last few years, complex network analysis has enabled a paradigm shift in the analysis of high energy heavy-ion collisions. Many novel techniques have been introduced to investigate the fractal structure and long-range relationships of time series. The term "fractal" was first used by Mandelbrot [2]. An index, which is termed as fractal dimension D, can be used to characterise fractal patterns. Such a fractal dimension has an numerous application in several fields, including medicine, human physiology, etc. Fractal dimension D is introduced, by which we can get information regarding experimental datasets. Knowledge of the multifractal spectrum is, in theory, totally similar to knowledge of the stochastic process. In case self-similar time series, the relationship between the Hurst exponent H and the fractal dimension D can be expressed as $D = 2 - H$. This Hurst exponent (H) is used to quantify the smoothness of fractal objects. If the value of $H > 0.5$, then it indicates the persistency of the time series and if $H < 0.5$, then it is called anti-persistent. In this analysis we have used experimental datasets from $^{16}O - Ag/Br$ interactions at 60 AGeV [3] and we compared the experimental findings with the Monte Carlo simulated event samples.

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Method of Analysis

Arold Edwin H. Hurst introduced the rescale range analysis method[4], which is the most well-known as well as oldest method for the study of complex network analysis. The details analysis are given in ref [4]. Here, we just state the important relation from the R/S analysis method. The entirety of the statistical information can be summed up in the five steps that are given in detailed below:

- Consider a set of data with N nodes, each of which is represented as $X_N = (x_i)$. From these nodes N, a sub–series with M number of nodes is defined such as $Y_M = (y_j)$, where $M = sN$, and $s \in (0, 1)$.
- Then, the mean of this sub–series is calculated using the expression $\bar{y_s} = \frac{1}{M}\sum_{k=1}^{M} y_k$.
- Create a cumulative data series of the partial summations $z_i = \sum_{k=1}^{i} y_k \sim \bar{y_s}$ *where* $i = 1, 2, \ldots, M$.
- The range can then be obtained as $R_s =_{max} z_i - {}_{min} z_i$.

- The range is rescaled by the standard deviation σ_s, such as $(R/S)_s = \frac{R_s}{\sigma_s}$. where the sample standard deviation σ_s is given by

$$\sigma_s = \left[\frac{1}{M}\sum_{k=1}^{M}(y_k \sim \bar{y_s})^2\right]^{\frac{1}{2}}$$

Discussions

In order to investigate the fractal behaviour of the multi-particle production dynamics, the rescaled range method is applied to the experimental datasets of $^{16}O - Ag/Br$ interactions at 60 AGeV.

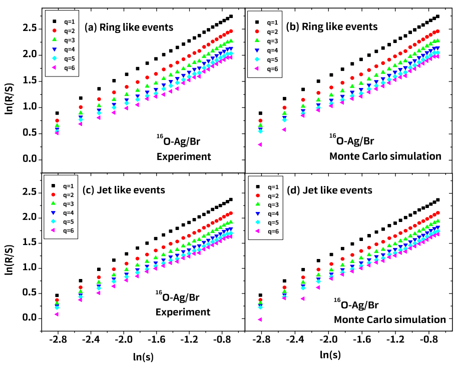

FIG. 1: Variations of ln(R/S) with ln(s) for ring–like and jet–like events

In addition to the experimental data, the equivalent Monte Carlo simulation data is also being examined and compared. In this case, the rescaled range was calculated using a large number of randomly selected subsamples with scales s ranging from 0.06 to 0.5 with an interval of 0.01.

The rescaled range (R/S) is calculated for a variety of scale s for a specified q-norm of a randomly selected subsample of events. Then, in that case, it is averaged over a sub-sample. Such calculations are performed for each event and then averaged across all events. Then the logarithm of rescaled range $ln(R/S)$ are drawn against ln(s), which is shown in Fig. 1. In Fig. 2, we have plotted the variation of

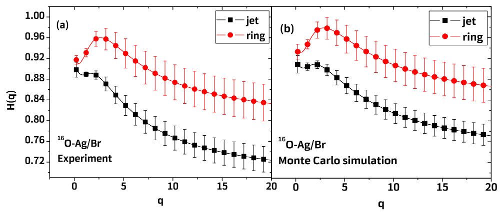

FIG. 2: Variation of Hurst exponent (H_q) with respect to q

$H(q)$ vs. q for both the ring–like and jet–like events in case of $^{16}O - Ag/Br$ interactions at 60 A GeV for both experimental as well as Monte Carlo simulated data respectively. From the figure, it is evident that the ring like events is more multifractal compare to the jet like events for both experimental and MC simulated data. From the study, it has been found that for $^{16}O - Ag/Br$ interactions at 60 AGeV the ring–like events shows more persistency compare to the jet–like events for both the experimental and Monte Carlo simulated datasets.

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