Relativistic hydrodynamics from Boltzmann equation in extended relaxation time approximation

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Boltzmann equation in the relaxation-time approximation [1] has been employed extensively to formulate relativistic dissipative hydrodynamics. In this proceedings, we outline our proposed framework for consistent derivation of first order relativistic dissipative hydrodynamics from the Boltzmann equation when the relaxation time depends on particle energy/momentum. We refer to this new proposed framework as the extended relaxation-time approximation (ERTA) [2]. This energy/momentum dependence of the relaxation-time leads to modification of the transport coefficients. We also study a number of intriguing scaling aspects for ratios of these transport coefficients.

We start with Boltzmann equation in ERTA, which was proposed earlier as [3]:

$$p^\mu \partial_\mu f = -\frac{(u\cdot p)}{\tau_{\rm R}(x,p)}\left(f - f^*_{\rm eq}\right), \qquad (1)$$

where p^μ is the particle four-momenta, u^μ is the fluid four-velocity and $\tau_{\rm R}(x,p)$ is the relaxation time. In the above equation, we define $f^*_{\rm eq} \equiv \left[\exp(\beta^*(u^*\cdot p) - \alpha^*) + a\right]^{-1}$ as the local equilibrium distribution function with $\beta^* \equiv 1/T^*$, $\alpha^* \equiv \mu^*/T^*$. Here, $a = 0,\, 1,\, -1$ for Maxwell Boltzmann (MB), Fermi Dirac (FD) and Bose Einstein (BE) statistics, respectively. The thermodynamic equilibrium quantities T^* and μ^* are defined in the local rest frame of $u^*_\mu = (1, 0, 0, 0)$ which we refer to as the thermodynamic rest frame.

On the other hand, the fluid velocity u^μ, temperature T and chemical potential μ are auxiliary hydrodynamic fields for an out of equilibrium system which are defined using the Landau frame and matching conditions [4]. We further introduce another local equilibrium distribution function $f_{\rm eq} \equiv \left[\exp(\beta(u\cdot p) - \alpha) + a\right]^{-1}$, where T and μ are defined in the fluid rest frame, $u^\mu = (1, 0, 0, 0)$. Implementing Chapman-Enskog like expansion around this equilibrium distribution, we derive the first-order gradient correction as

$$\delta f_{(1)} = \delta f_* - \frac{\tau_{\rm R}(x,p)}{(u\cdot p)}\, p^\mu \partial_\mu f_{\rm eq}, \qquad (2)$$

where, $\delta f_* \equiv f^*_{\rm eq} - f_{\rm eq}$ is a gradient correction to $\delta f_{(1)}$, arising due to the difference of the thermodynamic and hydrodynamic frame, and vanishes for a fluid in equilibrium.

To obtain the expression for δf_*, we relate the thermodynamic variables u^*_μ, T^* and μ^* to corresponding auxiliary hydrodynamic variables u_μ, T and μ as,

$$u^*_\mu = u_\mu + \delta u_\mu,\; T^* = T + \delta T,\; \mu^* = \mu + \delta\mu. \quad (3)$$

Taylor expansion of the equilibrium distribution $f^*_{\rm eq}$ about u^μ, T and μ up to first order in gradients leads to

$$\delta f_* = \left[-\frac{p_\mu \delta u^\mu}{T} + \frac{(u\cdot p - \mu)\delta T}{T^2} + \frac{\delta\mu}{T}\right] f_{\rm eq}\tilde{f}_{\rm eq}, \qquad (4)$$

where $\tilde{f}_{\rm eq} \equiv 1 - a f_{\rm eq}$. Imposing Landau frame condition, $u_\nu T^{\mu\nu} = \mathcal{E} u^\mu$ and the matching conditions, $\mathcal{E} = \mathcal{E}_{\rm eq}$ and $n = n_{\rm eq}$, we obtain

$$\delta u^\mu = \mathcal{C}_1 \frac{(\nabla^\mu \alpha)}{T}\,,\;\; \delta T = \mathcal{C}_2\,\theta\,,\;\; \delta\mu = \mathcal{C}_3\,\theta\,. \quad (5)$$

More information about the dimensionless variables $\mathcal{C}_1, \mathcal{C}_2$ and $\mathcal{C}_3$ can be found in Ref. [2].

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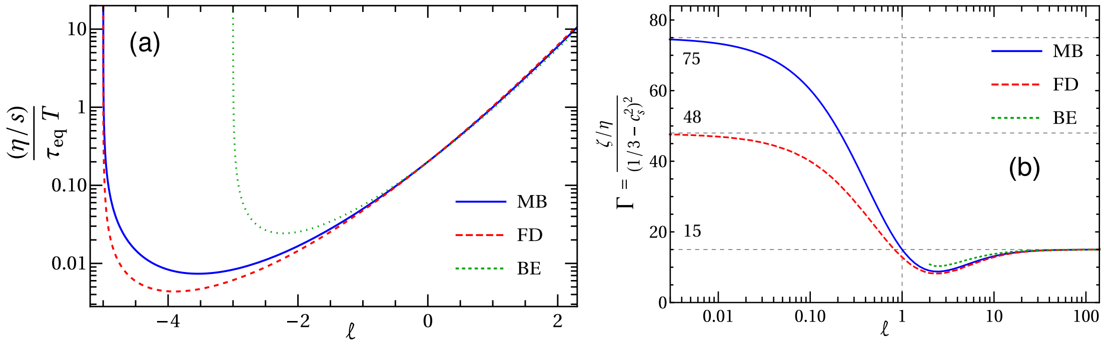

FIG. 1: (a) Dependence of $\eta/(s\tau_{\rm eq}T)$ with ℓ for mass-less and charge-less system; (b) Dependence of Γ with ℓ for massive and charge-less system. Figures adapted from [2].

We note that, for relaxation time independent of particle energy/momentum, the coefficients $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ vanishes. This, in turn, leads to vanishing of δu^μ, δT, $\delta\mu$ and hence the ERTA framework reduces to the usual Anderson-Witting RTA.

For the relaxation-time, we consider a power-law parametrization of the form

$$\tau_{\rm R}(x,p) = \tau_{\rm eq}(x)\left(\frac{u\cdot p}{T}\right)^{\ell},$$

where $\tau_{\rm eq}(x)$ is the energy/momentum independent part of relaxation time and ℓ represents the exponent [5]. With this, the relativistic Navier-Stokes expression for dissipative quantities are obtained as,

$$\pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu}\,,\, \Pi = -\zeta\,\theta\,,\, n^\mu = \kappa_n \nabla^\mu \alpha.$$

where the transport coefficients are given by,

$$\eta = \frac{K^+_{3,2}}{T},\;\; \zeta = -\mathcal{C}_2\left(\frac{\mathcal{E}+\mathcal{P}-\mu n}{T}\right) - \mathcal{C}_3 n + y^+_{3,1},$$
$$\kappa_n = \mathcal{C}_1\,\frac{n}{T} + \left(\frac{n}{\mathcal{E}+\mathcal{P}}\right) K^-_{2,1} - K^+_{1,1}\,. \qquad (6)$$

The detailed expressions are given in Ref. [2].

The specific viscosity for conformal chargeless system is shown in the Fig. 1(a) for MB, FD and BE statistics. We observe a minimum value of specific viscosity for negative ℓ in all three cases. It increases with ℓ and becomes statistics independent for $\ell > 0$. In Fig. 1(b), the scaling behaviour of the ratio of viscous coefficients ζ/η with square of conformality measure in small m/T limit is shown as a function of ℓ. A non-monotonic behaviour is observed with minima at $\ell \approx 2.5$ and a saturation $\Gamma \to 15$ is seen at large ℓ for all statistics.

We outlined the derivation of first-order dissipative hydrodynamic equations using an 'extended' relaxation time approximation for the Boltzmann equation, with energy/momentum dependent relaxation time. The associated transport coefficients are studied and the scaling behavior of their ratios are demonstrated for a power law parametrization. The presented framework has several applications in the context of hydrodynamic modeling of relativistic heavy ion collisions [2].

References

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