STUDIES IN NUCLEAR SCATTERING USING MANNING-ROSEN POTENTIAL

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Introduction

The primary aim of the scattering theory [1] is to understand an object by throwing something at it. By analyzing how that something bounces off, one can gather information about the object itself. The object that one wants to understand is the potential that exists between the two particles. In a scattering experiment a free particle or a beam of particles with known characteristic impinges upon a target particle, interact with it, and becomes scattered into a modified free state. One then observes the compatible characteristics of the scattered beam. Within the framework of non-relativistic Hamiltonian scattering theory, one tries to get solution of Schrödinger equation subject to proper boundary conditions and compute the differential cross-section $\sigma(\theta,\phi)$.

The collision of two particles having masses m_1 and m_2 is treated in the centre of mass system as the scattering of a particle with the reduced mass $M = m_1 m_2/(m_1+m_2)$ on a fixed scattering centre. The appropriate dynamical equation with which we are concerned here is the time-independent Schrödinger equation for a potential $V(\bar{r})$

$$\frac{\hbar^2}{2M}\left[\nabla^2 + k^2\right]\psi(\bar{r}) = V(\bar{r})\psi(\bar{r})$$

with the energy $E = \hbar^2 k^2/2M \rangle 0$. We now focus our attention to a particular solution of the above equation, denoted by $\psi^{(+)}$ such that it obeys asymptotic boundary condition

$$\psi^{(+)}(\bar{r}) \underset{r\to\infty}{\to} e^{ikz} + A(\theta,\phi)e^{ikr}/r .$$

To compute the scattering cross-section it is noted that the incident flux I_0 represents the probability current density of the incident beam e^{ikz} and is given by $I_0 = \hbar k/M$. The scattered flux, on the other hand, is computed from the radial component of the probability current associated with scattered beam $A(\theta,\phi)e^{ikr}/r$. This flux per unit solid angle expressed as $I(\theta,\phi) = \hbar k/M\left|A(\theta,\phi)\right|^2$. According to the definition of differential scattering cross-section we have the simple relation $\sigma(\theta,\phi) = \left|A(\theta,\phi)\right|^2$. The amplitude $A(\theta,\phi)$ is called the scattering amplitude.

The Schrödinger equation, is exactly solvable for a limited number of potentials like hydrogen atom, harmonic oscillator etc. at all energies and angular momentum [1]. However, to solve several other exponential types of potentials [2], one needs to resort to approximate treatments. The Manning-Rosen potential [3], frequently applied in molecular dynamics, has earlier been advocated by a number of researchers for bound as well as scattering state solutions. The results of these works have been applied quite successfully in atomic and molecular domain. However, we have applied this potential to the field of nuclear scattering theory where we consider its application on nucleon-nucleon and nucleon-nucleus systems.

According to the theory of second-order ordinary differential equation the solution which vanishes at the origin is regular and which does not is irregular. The solution satisfying the irregular boundary condition is called irregular or Jost solution namely $f_\pm(k,r)$. The Jost function approach is of importance to have its ability for dual treatments of bound and scattering states. Zeros of the Jost function for $\mathrm{Im}\,k > 0$ reproduce bound state energies [1] while negative of the phase of the Jost function gives the scattering phase shift. Physically, the Jost functions (on-and off-shell) represent the wave amplitudes. Also knowing the physical properties, one can reconstruct the potential that reproduces the observed phenomena. The Jost solution for the off-shell scattering satisfies the asymptotic boundary condition [1] $f_\ell(k,q,r) \xrightarrow[r\to\infty]{} e^{iqr}$. Both the off- and on-shell Jost functions have their two integral representations in terms of the irregular and regular solutions of the concerned potential. Thus, one can easily exploit them to calculate the off-shell Jost function.

It is a well known fact that the matrix element $T(p,q,k^2)$ is a function of three momentum variables p,q and k and is said to be on the energy shell when all of them are same. When two of the momenta are same, the matrix element is called half-shell one and when all the three momentum variables are unequal, the T-matrix element is completely off-shell. It is customary to define that the T-matrix has an importance in nucleon-nucleon dynamics as it is intimately connected to the experimentation.

Works in the dissertation

In this dissertation we have constructed regular solution for the Manning–Rosen potential via the differential equation approach to the problem and identify the irregular/Jost solution by exploiting the relation that exists between regular and irregular solutions[1]. Chapter 2 is devoted to compute S-wave scattering phase parameters for nucleon-nucleon and nucleon-nucleus elastic scattering and found good agreement in low energy scattering parameters **(J. Phys. G (2019) 115104, DOI: 10.1088/1361-6471/ab4118)**. The results for higher partial waves are presented in Chapter 3 **(Ann. Phys. 412 (2019) 168044, DOI: 10.1016/j.aop.2019.168044)**.

To study the charged hadron systems one has to add an electromagnetic potential which takes care of the charges to the nuclear part. Using Manning-Rosen plus electromagnetic interaction we have computed scattering phase shifts, by proper utilization of the phase function method (PFM) [4], for the nucleon-nucleon and the alpha-nucleon systems **(Commun. Theor. Phys. 72 (2020) 075301, DOI: 10.1088/1572-9494/ab8a1a)** in Chapter 4.

Also by exploiting the integral representation of the off-shell Jost function we have derived an expression for the same and thereby the half-shell T-matrix in maximal reduced form in Chapter 5. The off-shell contribution of the transition matrix is more appealing to various physical procedures. We have presented some model calculations to judge the merits of our expressions **(Phys. Scr. 95 (2020) 075308, DOI: 10.1088/1402-4896/ab95ae)**.

The transition matrices provide a suitable starting point for a variety of calculations like nuclear reaction processes and nuclear matter binding energies. In the Chapter 6 we have constructed the off-shell solutions for the Manning-Rosen potential by exploiting the ordinary differential equation method together with the properties of the special functions of mathematics. Making use of the off-shell solutions fully off-shell T-matrix has been calculated by using an expression which does not involve the potential explicitly and made them amenable to numerical treatment **(Few-Body Syst. (2021) DOI: 10.1007/s00601-021-01603-6) & (Pramana- J. Phys. DOI:10.1007/s12043-021-02206-w).**

Conclusion

We have parameterized the Manning-Rosen potential for real nuclear systems and noticed that excellent results with regard to bound and scattering states observables have been produced. In principle, the (n-d) system is generally treated within the three-body model of interaction. In contrast, we have applied the two-body model of interaction and achieved excellent agreement in phase shift values with the sophisticated calculations. Thus, our three parameter nuclear Manning-Rosen potential has the ability to reproduce the various physical observables for nuclear systems and provides hopeful avenue for the theoretical physicists.

References

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