p-d scattering for the q-deformed Hulthén potential

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Introduction

The Hulthén potential [1] is an exponential-type potential that is exactly solvable for the S wave. Physicists obtained higher partial wave solutions of the Schrödinger equation using approximation prescribed by Green and Aldrich to the centrifugal barrier term for this Potential. Hall, Saad, and Sen recently found the exact normalized eigenfunctions for the q-deformed Hulthén potential [2] and the solutions for the more generalized deformed Hulthén potential with the Crum - Darboux transformation. The q-deformed Hulthén potential allows for a more comprehensive understanding of the dynamics involved in charged hadron scattering, providing insights into the underlying physics at play. Recently, Majumder et al. [3], by introducing the concept of the energy-dependent correction factor, incorporated the q-deformed Hulthén potential in the nuclear domain and found the scattering phase shifts of alpha-p and alpha-n systems through the phase function method without solving the Schrödinger equation. The all-partial wave solution for this combined interaction is not available in the literature. Here, we obtained a regular solution for the Schrödinger equation using the q-deformed Hulthén nuclear plus the q-deformed Hulthén atomic potential without considering the energy-dependent correction factor. Obtained the Jost function and Jost solution. Finally we have computed the scattering phase shift of the proton-deuteron system using the Jost function technique.

Methodology

The Schrödinger equation for a particle moving in the combined interaction field of q-deformed Hulthén atomic potential and q-deformed Hulthén nuclear potential reads as

$$\left[\frac{d^2}{dr^2} + k^2 - V_{ND}(r) - V_{AD}(r) - \frac{l(l+1)}{r^2}\right]\phi_{DD}(k,r) = 0\,, \quad (1)$$

where $$V_{ND}(r) = \frac{V_{ON}e^{-\delta r}}{(1-qe^{-\delta r})} \quad (2)$$

is the q-deformed nuclear Hulthén potential and

$$V_{AD}(r) = \frac{V_{OA}e^{-\delta r}}{(1-qe^{-\delta r})}. \quad (3)$$

is q-deformed nuclear potential. Here V_{ND} and V_{AD} are the strengths of the respective potentials. Eq. 1 does not admit exact analytical solutions due to a centrifugal barrier factor. We have adjusted the centrifugal barrier as in Ref. [2] to get the solution. Using V_{ND} and V_{AD} along with the modified centrifugal term, Eq. 1 is rewritten as

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{V_{eff}e^{-\delta r}}{(1-qe^{-\delta r})} - \frac{\delta^2 l(l+1)qe^{-\delta r}}{(1-qe^{-\delta r})^2}\right]\phi_{DD}(k,r) = 0. \quad (4)$$

Where $V_{eff} = V_{ON} + V_{OA}$ is the effective strength. We consider the same range approximation scheme and express the inverse range by δ. On use of the following transformation

$$\phi_{DD}(k,r) = \frac{1}{\delta^{l+1}}\left(1 - qe^{-\delta r}\right)^{\eta}\left(qe^{-\delta r}\right)^{\gamma} g_{DD}(k,r) \quad (5)$$

along with $\left(1 - qe^{-\delta r}\right) = z$, Eq. 5 reduces to the standard form of Gaussian hypergeometric equation [6] in the condition

$$(\gamma^2 + k^2/\delta^2)\frac{z}{1-z}g_{DD}(z) + \left\{\left(\eta(\eta-1) - l(l+1)\right)\right\}\frac{1}{z}g_{DD}(z) = 0 \quad (6)$$

which yields $\eta = l+1$ or $-l$ and $\gamma = \pm\frac{ik}{\delta}$. Inserting the exact solution of Eq. (6), which is a Gaussian hypergeometric function, into Eq. (5) with $\eta = l+1$ and $\gamma = -\frac{ik}{\delta}$, one obtains the regular solution of the combined interaction. The Jost function is obtained using the regular solution and some standard relation and transformation

*Electronic address: *n.dibakar2001@gmail.com* equation of the Gaussian Hypergeometric function [7], which is expressed as

$$f_{DD}(k) = \frac{\Gamma(2l+2)\Gamma\left(1-\frac{2ik}{\delta}\right)}{\Gamma\left(l+1-\frac{ik}{\delta}-P\right)\Gamma\left(l+1-\frac{ik}{\delta}+P\right)\delta^{l}} \qquad (7)$$

Results and discussion

For the computation, we have used the phase shift expression $\delta_p = -\tan^{-1}\left[\frac{Imag(f_{DD}(k))}{Real(f_{DD}(k))}\right]$, along with the value $\frac{\hbar^2}{2\mu} = 31.1025$ MeV fm^2 and $\frac{V_{OA}}{\delta} = 0.04629$ fm^{-1} for the p-d system. Here, 'q' is the modifying quantity with the condition log(q)/δ < r < ∞, which reduces to one for the standard Hulthén potential. Hence, the value of q should always lies between 0 and 1. Since $^2S_{1/2}$ is bound state at an energy E_B=7.718 MeV, we obtained the parameter values by fixing the binding energy. The best-fit parameters are obtained through the R-square fitting method and are noted in Table 1.

Table 1: Best-fit parameters value

State	V1 (fm^{-2})	δ (fm^{-1})	q
$^2S_{1/2}$	-1.5933	0.8217	0.9340
$^2P_{1/2}$	-0.2881	0.3638	0.8137
$^2P_{3/2}$	-0.6687	1.31	0.0481

The phase shift obtained by substituting the best-fitted parameters in Eq. (7) agrees with the data of Huttel et al. [4], as portrayed in Fig.1. The goodness of fit for the three different states has been mentioned in Table 2.

Table 1: Goodness of fit

State	SSE	R^2	Adj R^2	RMSE
$^2S_{1/2}$	0.9083	0.9822	0.9645	0.9531
$^2P_{1/2}$	0.0183	0.9994	0.9994	0.0957
$^2P_{3/2}$	0.4147	0.9623	0.9246	0.6440

The small deviations observed in the phase shift might have arisen due to the non-inclusion of the spin-orbit interaction and tensor potentials. We are considering it and are working on this presently.

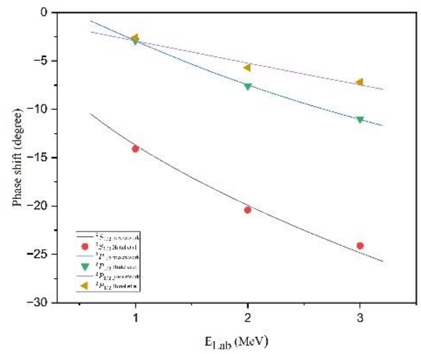

Fig. 1 Phase shift as a function of E_{Lab}

Conclusions

The results indicate that our methodology and the q-deformed Hulthén potential, both as atomic and nuclear potential, can reproduce the scattering phase shift of the p-d system. Hence, it can be used as a simplified central potential model to explore any charge-hadron scattering in low-energy regions.

References

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