Optimizing Giant Dipole Resonance Parameters using the Bayesian Inference Technique

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Introduction

The photoabsorption cross section (PACS) in the ground state of the nucleus is an important quantity utilized in different basic physics problems and applications. In recent years, it has attracted a lot of theoretical as well as experimental interests, in particular, to its relation to the dipole polarizability of the nucleus, which is connected to the nuclear equation of state. In addition, the PACS is directly related to the photon strength function, which is one of the key ingredients in nuclear reaction model calculation and (n,γ) capture rate calculation, which is crucial in determining the elemental abundance in nuclear astrophysics.

In the region $E_\gamma \approx$10-25 Mev, the PACS is primarily dominated by the isovector giant dipole resonance (GDR). It is described macroscopically as an out-of-phase vibration of proton and neutron fluids in the nucleus. It is mainly governed by three parameters, namely the centroid energy (E_G), the width (Γ_G) and the strength (S_G). A reliable information of these parameters is crucial for two primary purposes. First, it will verify different theoretical approaches that calculate these parameters, and second, it will examine their utilisation in nuclear reaction model calculation. Traditionally, the GDR parameters have been determined using the Brink-Axel Lorentzian[1][2]. In this work, we aim to optimize the GDR parameters in the simplified Lorentzian model using the Bayesian inference method. One of the primary advantages of this method is that it provides the correlations among the extracted parameters, which are very important for nuclei where the photo absorption data does not exist. We have performed the Bayesian analysis of the photoabsorption data for a wide range of nuclei. In this paper, we report the representative results for ^{148}Sm.

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Method

The GDR cross-section $\sigma(E_\gamma)$ is Lorentzian in shape and characterized by three parameters: centroid energy(E_G), resonance width(Γ_G) and strength function(S_G). The GDR cross-section is given by,

$$\sigma(E_\gamma) = \frac{2\sigma_{TRK}}{\pi} \frac{S_G \Gamma_G E_\gamma^2}{(E_\gamma^2 - E_G^2) + E_\gamma^2 \Gamma_G^2} \quad (1)$$

where, $\sigma_{TRK} = \frac{60NZ}{A}$ from Thomas-Reiche-Kuhn (TRK) sum rule. We get the joint posterior distributions of the parameters using Bayesian parameter estimation. From that, we can study the distribution of the parameters and the correlation among the parameters. This approach is based on the Bayes theorem,

$$P(\theta|D) = \frac{\mathcal{L}(D|\theta)P(\theta)}{\mathcal{Z}} \quad (2)$$

where θ and D are model parameters and experimental data, respectively. Our model is defined in Eq. (1) and model parameters (θ) are E_G, Γ_G, S_G. Here, D is the experimental cross-section data taken from EXFOR website [3]. $P(\theta|D)$ is the joint posterior distribution of the parameters, $\mathcal{L}(D|\theta)$ is the likelihood function, $P(\theta)$ is the prior value of the model parameters, and $\mathcal{Z}$ is the evidence value define as $\sum_\theta \mathcal{L}(D|\theta)P(\theta)$.

Results and Discussion

In the Bayesian parameter estimation, we always get the probability distributions of the parameters. From each probability distribution, we take the median of the distribution as the best-fit value and get an error of the parameter defined by 1σ deviation from the best-fit value. We also calculated the correlation between the parameters. In FIG.1, the joint posterior probability distributions are shown and in FIG.2, the best fit to the experimental PACS is presented with the optimum values of the GDR parameters obtained using the Bayesian technique.

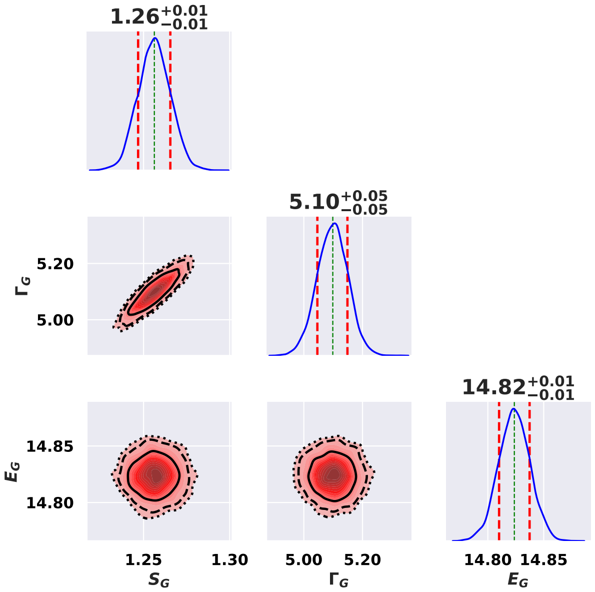

FIG. 1: Joint posterior probability distribution plot of the GDR parameters.

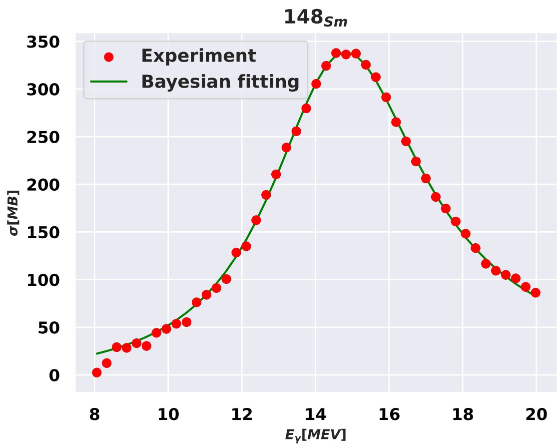

FIG. 2: Comparison between the experimental data and the result got after Bayesian fitting.

TABLE I: Pearson correlation coefficient (r) between GDR parameters for ^{148}Sm.

r[S_G,E_G]	r[Γ_G,E_G]	r[S_G,Γ_G]
-0.02	-0.03	0.86

From TABLE I, it is very clear that the correlation between S_G and Γ_G is very strong. Therefore, we cannot vary these two parameters independently.

TABLE II: The most probable value of GDR parameters got after Bayesian analysis for ^{148}Sm.

$\langle S_G \rangle$	$\langle \Gamma_G \rangle$	$\langle E_G \rangle$
1.26	5.10	14.82

According to the Goldhaber-Teller(GT) model [4], $E_G \propto A^{-\frac{1}{3}}$ and Steinwedel-Jesen(SJ) suggest [5] $E_G \propto A^{-\frac{1}{6}}$. However, it is observed that the mass dependence is somewhere intermediate between these two models, and it is given by [6],

$$E_G = 31.8A^{-\frac{1}{3}} + 20.6A^{-\frac{1}{6}} \tag{3}$$

TABLE III: Comparison between the values of E_G with bayesian analysis and theoretical model for ^{148}Sm.

Bayesian model value	Intermediate GT and SJ model value	fractional error
14.82	14.97	0.01

As TABLE III shows, our Bayesian results are quite close to the analytical form given in Eq. (3).

Conclusion

We have performed detailed Bayesian analysis of the experimental photoabsorption cross section for a wide range of nuclei to draw a general conclusion on the GDR parameters. The representative results for ^{148}Sm is presented in this paper. We found a strong correlation between the GDR width and strength function, restricting the independent variation of these parameters in future analysis. The details will be presented during the symposium.

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