# Excellent prediction of Two-neutron separation energies ($S_{2n}$) of some of the nuclides of Astrophysical interest by the simple Garvey-Kelson Extrapolation method


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## Introduction

The nucleosynthesis in stars has been studied in order to better understand the formation of elements as well as the abundance of some of the elements in the universe. These astrophysical processes, however, take place far from the valley of stability, where experimental masses are not known, and no theoretical model exists that can precisely predict these atomic masses. Therefore, in the absence of any experimental atomic mass measurements in this region and reliable model-based extrapolations, precise local extrapolations of the available experimental atomic mass data would be of great help in arriving at the atomic masses of nuclei far from the valley of stability. Local interpolations or extrapolations of the experimental mass values have been tried as an alternative and more plausible way to reach the nuclides far from the valley of stability. A great deal of work has been done to obtain formulas that will give the masses of all particle-stable nuclides in the periodic table. Such formulas, based usually on the masses of known nuclides, may be used to predict the masses of undiscovered isotopes. Garvey and Kelson presented a relation based on an independent particle model of the nucleus. These global mass formulas, however, will in general be too approximate to make predictions to a sufficient degree of accuracy. However, simple relations between neighbouring nuclidic masses was developed by Garvey and Kelson which will be valid, independently of the actual variation of mass with atomic number and charge. Such relations will not replace mass formulas, but may be very useful because of their simplicity and greater accuracy in making ground-state mass predictions. Here in this paper we tried to revisit the Garvey-Kelson relations [1] in order to give an impression of the validity of this approach to predict the $S_{2n}$, the two neutron separation energies of the most neutron-rich nuclides of Ga, Ge, As and Se and compared them with the extrapolated $S_{2n}$ values of AME20 [2] the most recent Atomic Mass Evaluation and with the most credible theoretical mass model predictions of FRDM [3,4], the Finite Range Droplet Model HFB-21[5].

## Method

In 1966 Garvey and Kelson presented such a relation based on an independent- particle model of the nucleus [1]:

$$M_{(N+2,\,Z-2)} - M_{(N,\,Z)} + M_{(N,\,Z-1)} - M_{(N+1,\,Z-2)} + M_{(N+1,\,Z)} - M_{(N+2,\,Z-1)} = 0 \quad \ldots. (1)$$

With five known masses, a sixth unknown one can be predicted. The applicability of the method has been tested with cases in which either all five mass excess values of Equ. (1) are known experimentally or have been estimated in AME20. Separation energies in MeV of particles or groups are obtained as the following combination of atomic masses from the calculated atomic mass data from the Garvey-Kelson prescription using the relation $S_{2n} = -M_{(A,\,Z)} + M_{(A-2,\,Z)} + 2n$ and are tabulated in column 2 as G-K in Table 2. Listing of all equations (according to Equ. 1) needed to calculate the mass excess values.

**Table 1: Listing of Garvey-Kelson equations**

| Isotope | Garvey-Kelson Relation |
|---|---|
| $^{85}$Ga | $^{85}$As + $^{84}$Ga - $^{84}$Ge + $^{86}$Ge - $^{86}$As |
| $^{86}$Ga | $^{86}$As + $^{85}$Ga - $^{85}$Ge + $^{87}$Ge - $^{87}$As |
| $^{87}$Ga | $^{87}$As + $^{86}$Ga - $^{86}$Ge + $^{88}$Ge - $^{88}$As |
| $^{88}$Ga | $^{88}$As + $^{87}$Ga - $^{87}$Ge + $^{89}$Ge - $^{89}$As |
| $^{89}$Ga | $^{89}$As + $^{88}$Ga - $^{88}$Ge + $^{90}$Ge - $^{90}$As |
| $^{90}$Ga | $^{90}$As + $^{89}$Ga - $^{89}$Ge + $^{91}$Ge - $^{91}$As |
| $^{91}$Ga | $^{91}$As + $^{90}$Ga - $^{90}$Ge + $^{92}$Ge - $^{92}$As |
| $^{88}$Ge | $^{88}$Se + $^{87}$Ge - $^{87}$As + $^{89}$As - $^{89}$Se |
| $^{89}$Ge | $^{89}$Se + $^{88}$Ge - $^{88}$As + $^{90}$As - $^{90}$Se |
| $^{90}$Ge | $^{90}$Se + $^{89}$Ge - $^{89}$As + $^{91}$As - $^{91}$Se |
| $^{91}$Ge | $^{91}$Se + $^{90}$Ge - $^{90}$As + $^{92}$As - $^{92}$Se |
| $^{92}$Ge | $^{92}$Se + $^{91}$Ge - $^{91}$As + $^{93}$As - $^{93}$Se |
| $^{93}$Ge | $^{93}$Se + $^{92}$Ge - $^{92}$As + $^{94}$As - $^{94}$Se |
| $^{94}$Ge | $^{94}$Se + $^{93}$Ge - $^{93}$As + $^{95}$As - $^{95}$Se |
| $^{87}$As | $^{87}$Br + $^{86}$As - $^{86}$Se + $^{88}$Se - $^{88}$Br |
| $^{88}$As | $^{88}$Br + $^{87}$As - $^{87}$Se + $^{89}$Se - $^{89}$Br |
| $^{89}$As | $^{89}$Br + $^{88}$As - $^{88}$Se + $^{90}$Se - $^{90}$Br |
| $^{90}$As | $^{90}$Br + $^{89}$As - $^{89}$Se + $^{91}$Se - $^{91}$Br |
| $^{91}$As | $^{91}$Br + $^{90}$As - $^{90}$Se + $^{92}$Se - $^{92}$Br |
| $^{92}$As | $^{92}$Br + $^{91}$As - $^{91}$Se + $^{93}$Se - $^{93}$Br |
| $^{93}$As | $^{93}$Br + $^{92}$As - $^{92}$Se + $^{94}$Se - $^{94}$Br |
| $^{93}$Se | $^{93}$Kr + $^{92}$Se - $^{92}$Br + $^{94}$Br - $^{94}$Kr |
| $^{94}$Se | $^{94}$Kr + $^{93}$Se - $^{93}$Br + $^{95}$Br - $^{95}$Kr |
| $^{95}$Se | $^{95}$Kr + $^{94}$Se - $^{94}$Br + $^{96}$Br - $^{96}$Kr |
| $^{98}$Se | $^{98}$Kr + $^{97}$Se - $^{97}$Br + $^{99}$Br - $^{99}$Kr |
| $^{99}$Se | $^{99}$Kr + $^{98}$Se - $^{98}$Br + $^{100}$Br - $^{100}$Kr |

**Table 2: Garvey-Kelson $S_{2n}$ predictions compared with AME20 predictions and FRDM and HFB21 mass model predictions**

| Iso-tope | Two-neutron separation energy values ($S_{2n}$) in (keV/$c^2$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | G-K | AME20 | GK-AM | FRDM | GK -FR | HFB -21 | GK -HF |
| $^{85}$Ga | 6571 | 6630 40 | 59 | 6570 | 1 | 6600 | 29 |
| $^{86}$Ga | 6034 | 5810 400 | 224 | 5980 | 54 | 5870 | 164 |
| $^{87}$Ga | 5579 | 5270 500 | 309 | 5510 | 69 | 5600 | 21 |
| $^{88}$Ga | 5032 | 4770 640 | 262 | 5050 | 18 | 5260 | 228 |
| $^{89}$Ga | 4705 | --- | -- | 4450 | 250 | 4840 | 135 |
| $^{90}$Ga | 4393 | --- | -- | 4030 | 363 | 4340 | 53 |
| $^{91}$Ga | 3838 | --- | -- | 4170 | 332 | 3740 | 98 |
| $^{88}$Ge | 6537 | 6260 590 | 277 | 6460 | 77 | 6270 | 267 |
| $^{89}$Ge | 5836 | 5590 500 | 246 | 6030 | 227 | 5790 | 46 |
| $^{90}$Ge | 5158 | 5090 640 | 68 | 5570 | 412 | 5560 | 402 |
| $^{91}$Ge | 4966 | --- | -- | 4900 | 66 | 4910 | 56 |
| $^{92}$Ge | 4620 | --- | -- | 4990 | 370 | 4680 | 60 |
| $^{93}$Ge | 4073 | --- | -- | 4530 | 457 | 4190 | 117 |
| $^{94}$Ge | 3282 | --- | -- | 3820 | 58 | 3780 | 498 |
| $^{89}$As | 7439 | 7060 300 | 379 | 7400 | 39 | 7310 | 129 |
| $^{90}$As | 6470 | 6680 450 | 210 | 7020 | 550 | 6940 | 470 |
| $^{91}$As | 6089 | 6110 500 | 21 | 6590 | 501 | 6660 | 571 |
| $^{92}$As | 5926 | 5530 640 | 396 | 6140 | 214 | 6010 | -84 |
| $^{93}$As | 5524 | --- | -- | 5780 | 256 | 5930 | 406 |
| $^{93}$Se | 6855 | 6420 590 | 435 | 7180 | 325 | 6900 | 45 |
| $^{94}$Se | 6512 | 6220 640 | 292 | 6820 | 308 | 6690 | 178 |
| $^{95}$Se | 5952 | 5740 640 | 212 | 6710 | 758 | 6530 | 578 |
| $^{96}$Se | 4569 | --- | -- | 5030 | 461 | 5140 | 571 |
| $^{99}$Se | 4663 | --- | -- | 4870 | 207 | 4900 | 237 |

## Results and discussion

It is interesting to note from the Table 2 that the G-K $S_{2n}$ predictions are very close to the most recent and very precise AME20 data within their uncertainties, indicating the predictive power of G-K relations in a limited context. Also the deviations of the G-K predictions from the HFB21, FRDM indicate that G-K predictions are very close to these two model predictions only deviating in the cases of highly neutron rich nuclides which can be easily understood as the mass data of nuclides used in the chain of the G-K formula involve large uncertainties. It is our feeling that it is a useful approximation and should be exploited in ground-state energy predictions.

## References


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