Chebyshev shape parametrization

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Introduction

Nuclear structural and dynamic studies play a crucial role in understanding collective phenomena within atomic nuclei. To gain insight into a nucleus's structural properties, it is essential to explore nuclear shape parametrization and the total potential energy as a function of its deformation. Over the past decades, several shape parametrization methods have been developed, including the traditional Bohr-Wheeler approach (using β and γ parameters), Cassini ovaloid, Funny-Hills, TKS, and Fourier shape parametrizations [1, 2]. Despite these advancements, determining the precise structure of the nucleus remains an open question in nuclear physics. This study presents a new approach to describe nuclear shapes, termed the Chebyshev shape parametrization. The profile function of the nuclear shape is constructed using the Chebyshev series [3]. The total deformation energy of the system, based on the Chebyshev parametrization, is calculated using the Lublin-Strasbourg Drop (LSD)model [4]. The model results are validated by comparing with other existing shape parametrizations.

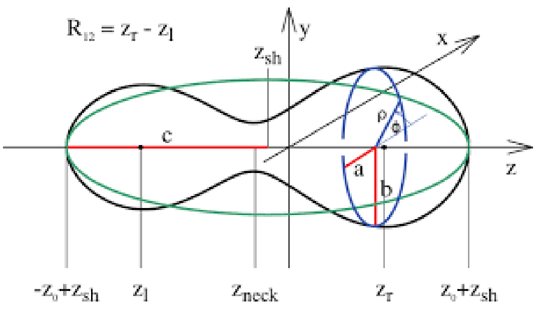

FIG. 1: Schematic diagram of nuclear shape using Chebyshev shape parametrization.

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Theoretical Background

We employed Chebyshev polynomials of the first kind, $T_n(x)$, to develop the nuclear shape parametrization. These polynomials can generate complex geometric shapes with high precision and offer a more realistic representation without boundary constraints. The resulting profile function of the system is expressed as follows [5],

$$\frac{\rho_s^2(u)}{R_0^2} = \sum_{n=0}^{\infty} a_n T_n(u), \quad (1)$$

here, $\rho_s(u)$ is the perpendicular distance from the surface to the symmetry axis, and R_0 is the radius of the spherical nucleus. a_n is the Chebyshev series coefficients corresponding to the deformation parameters. In this work, we used three deformation parameters: a_2, which primarily governs elongation, and a_4, which affects neck formation, with both being interdependent. The parameter a_3 represents asymmetry in the nuclear shape. The nucleus is traditionally modelled as an incompressible liquid drop with constant volume. Therefore, to accurately describe the complex geometry of the nucleus, we incorporate volume conservation and centre of mass formalism, linking the deformation parameters to the Chebyshev series coefficients. We employ the LSD model to determine the total potential energy of the system. This cumulative potential energy encompasses distinct components: surface energy, Coulombic energy, rotational energy, curvature energy, and congruence energy terms. The potential energy surface (PES) is formally defined as aggregating these energy contributions, each representing specific aspects and interactions within the system under consideration. Total potential energy is

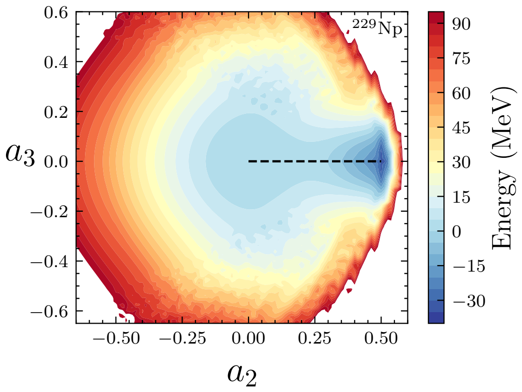

FIG. 2: Cross sections of PES of ^{229}Np in the two-dimensional deformation subspace defined by (a_2, a_3).

defined as,

$$\begin{aligned} V(q)_{LSD+C} = & E_S^{(0)}(B_S(a_i) - 1) \\ & + E_C^{(0)}(B_C(a_i) - 1) \\ & + E_r^{(0)}(L^2 B_J(a_i) + K^2 B_K(a_i)) \\ & + E_{curv}^{(0)}(B_{curv}(a_i) - 1) \\ & + E_{cong}^{(0)}(B_{cong}(a_i) - 1) \, . \end{aligned} \quad (2)$$

$B_S, B_C, B_J, B_K, B_{curv}$ and B_{cong} denote the deformation-dependent energy coefficients linked to surface, Coulombic, rotational, curvature, and congruence energies, respectively. These coefficients, reliant on the deformation of the system, are derived from the Chebyshev shape parametrization. Conversely, $E_S^{(0)}, E_C^{(0)}, E_r^{(0)}, E_{curv}^{(0)}$ and $E_{cong}^{(0)}$ represent energies associated with the spherical nuclei.

Results and Discussion

In this work, we have investigated the PES of ^{229}Np isotope at high energy fission limits. The shell correction and pairing vanish at high energy fission limits. Hence, we consider only macroscopic models (LSD model) to calculate the total potential energy of the nucleus. Fig. 2 represents cross section of PES of ^{229}Np in the two-dimensional deformation subspace defined by (a_2, a_3) at angular momentum, $L = 0$, estimated using LSD with congruence energy. The mean path to symmetric fission $(a_3 = 0)$ is represented using a dashed line along a_2. As the figure shows, the asymmetric fission mode is also possible for this particular nuclei.

In Summary, we have introduced a Chebyshev shape parametrization for defining the complex shapes of the nucleus. We calculated the deformation-depended energy coefficient and plotted PES of the ^{229}Np nucleus. The analysis of the PES enabled us to successfully predict different fission modes.

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