Study of proton-neutron entanglement entropy using nuclear shell model

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Introduction

The atomic nucleus consists of protons and neutrons, held together by strong nuclear force. The nuclear system is a type of quantum many-body problem, which can be solved using configuration-interaction methods. The nuclear shell model approach is one of the successful ones across different mass regions. As the number of nucleons increases, the size of the Hilbert space increases exponentially giving rise to computational challenges. Quantum computers can solve this problem using various quantum algorithms developed in the past few years [1–4]. Recently, quantum information has gained popularity in studying the properties of nuclear structure by calculating entanglement entropy, mutual information, and quantum relative entropies [5, 6]. This work investigates the proton-neutron entanglement entropy of a few nuclei in the *sd*-shell.

Formalism

Entanglement is a measure of dependence between different systems or different partitions within the same system; greater dependence implies higher entanglement between those systems. The atomic nucleus provides a natural bi-partition where protons and neutrons can be considered as two different subsystems. For such a system, the basis is factorized into proton (p) and neutron (n) components and represented as $|\alpha\rangle = |\mu_p\rangle \otimes |\sigma_n\rangle$ [5, 6]. For this work, we have utilized the von Neumann entropy S = -tr $\rho \log \rho$ of few selected *sd*-shell nuclei with USDB interaction. In calculating the von Neumann entropy, the total density matrix $\rho_{\mu'\sigma',\mu\sigma} = c_{\mu'\sigma'} c^*_{\mu\sigma}$ is traced over one of the subspace indices. The wavefunction to produce the density matrix is represented by

$$|\psi\rangle = \sum_{\mu,\sigma} c_{\mu,\sigma} |\mu_p\rangle \otimes |\sigma_n\rangle, \tag{1}$$

where the amplitudes are separable $c_{\mu,\sigma} = a_\mu b_\sigma$ if the proton-neutron states are unentangled. The reduced density matrix $\rho^{red}_{\mu'\mu} = \sum_\sigma c_{\mu'\sigma} c^*_{\mu\sigma}$ can be now be used to calculate

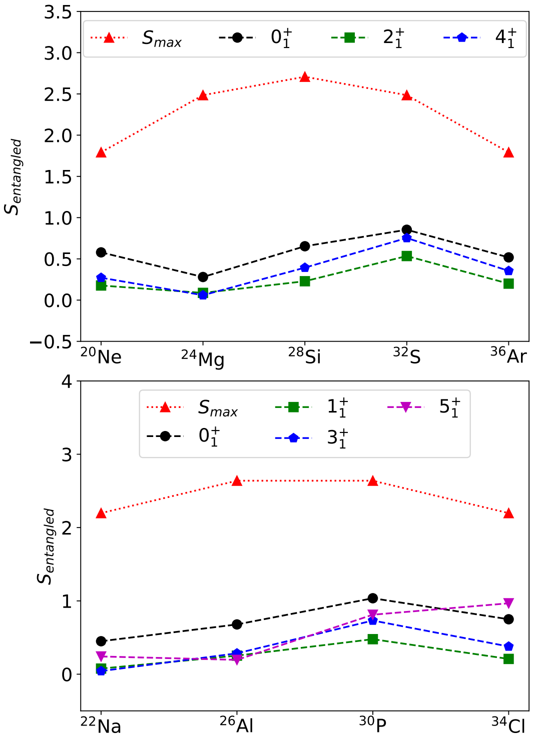

FIG. 1: Entanglement entropy of $N = Z$ nuclei across *sd*-shell.

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entanglement entropy

$$S_{entangled} = -tr\rho^{red} \log \rho^{red}. \qquad (2)$$

Another quantity we are interested in this work is the maximum entanglement entropy (S_{max}) which is defined as:

$$S_{max} = \ln(\min(dim_{\pi}, dim_{\nu})) \qquad (3)$$

Results and Discussions

In this work, we have first calculated the entanglement entropy for the even-even nuclei ranging from $A = 20$ to 36 corresponding to USDB interaction. All the calculations are performed in J-scheme. The primary objective is to observe the proton-neutron entanglement pattern of 0_1^+-2_1^+-4_1^+ states and how it changes from ^{20}Ne to ^{36}Ar. All the states follow a similar trend; entropy peaks at A=32 and drops again at A=36, as shown in the first panel of Fig. 1. The ground state for all the nuclei has higher entropy than their excited states. In the next case calculations for odd-odd nuclei ranging from A=22 to 34 were performed using the same interaction. In this case, the entropy for the states 0_1^+-1_1^+-3_1^+ follow a similar trend as even-even nuclei, where the entropy peaks at A=30 and drops at A=34. But the 5_1^+ states follow an unusual pattern, it peaks at A=34. The ground states for the odd-odd nuclei have lower entropy than their excited states, with A=34 being an exception, as shown in the second panel of Fig.1.

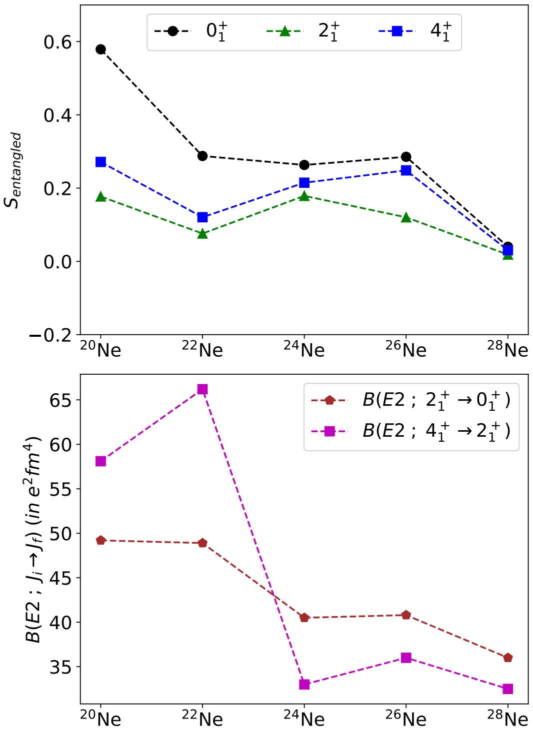

FIG. 2: Entanglement entropy for Ne chain and corresponding *B(E2)* values.

We have also studied the entanglement patterns for a chain of Ne nuclei ranging from ^{20}Ne to neutron-rich ^{28}Ne, our main focus is on the 0_1^+-2_1^+-4_1^+ states for this chain. The overall trend for all three states of the Ne chain shows a decrease in entanglement entropy as the number of neutrons increases as shown in the first panel of Fig. 2. Additionally, we showed the $E2$ transition strengths: $B(E2; 2^+ \to 0^+)$ and $B(E2; 4^+ \to 2^+)$ for the same sets of nuclei in the second panel of Fig. 2 and observed an overall decreasing pattern just like proton-neutron entanglement entropies.

In conclusion, the proton-neutron entanglement entropy, in combination with other entanglement measures, can provide valuable information about collectivity within an isotopic chain. It can also provide information related to nuclear interaction.

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