Configuration-constrained cranked shell model analysis of the odd-Z nuclei 171,173,175 Re

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Introduction

The odd-Z isotopes offer a unique opportunity to investigate the high-spin structures of rare-earth nuclei, which have drawn significant attention due to a spectrum with increased level density. An extra nucleon can polarize the core, affecting its motion. Also, the motion of the core can change the dynamics of the extra nucleon. Determining spin-parity and configuration assignments for states in odd-A nuclei is challenging. Experimental observations have identified several one- and three-quasiparticle bands in the odd-Z nuclei ^{171,173,175}Re. A configuration-constrained Cranked Shell Model (CSM) study of spins, parities, band crossing frequencies, and alignment gains offers a valuable approach to analyzing the various bands in these nuclei.

In this work, to study the behavior of various bands in ¹⁷⁵Re, Total Routhian Surface (TRS) calculations are done to extract the nuclear deformation parameters as a function of $\hbar\omega$. The angular momentum alignments and the dynamic moment of inertia are calculated in the framework of CSM by extending the approach in Ref. [1, 2, 3] similar to the one in Ref. [4, 5]. The configurations are assigned to bands with the help of the quasiparticle routhians, which represent the energy of nuclear states in a rotating frame.

Formalism

The CSM has been highly successful in describing near-yrast high-spin states in nuclei. The CSM Hamiltonian can be written as [1]

$$H_{\omega} = H_0 - \omega J_x, \qquad (1)$$

where H_{ω} is the Hamiltonian of the particle in the rotating frame and H_0 is the Hamiltonian which describes the sum of the smooth liquid drop energy and the oscillatory part representing the strength of the quantum effects in a fixed frame. J_x is the projection of the angular momentum of the particle onto the rotation axis. The eigenstates of H_{ω} can be written as

$$E_{\omega} = E_{LDM} + \sum_{Z,N} \delta E - \omega J_x.$$
 (2)

The term E_{LDM} represents the liquid drop energy corresponding to the triaxially deformed nucleus. δE represents the shell correction and the pairing energy. For the calculations, we utilize a realistic mean field of the triaxial Woods-Saxon (WS) potential [2, 3], using universal parametrization, expressed as a sum of central mean-field potential (V_{WS}), the Coulomb interaction term (V_C), and the spinorbit potential (V_{SO}), given by

$$V(\vec{r},\beta) = V_{WS}(\vec{r},\beta) + V_C(\vec{r},\beta) + V_{SO}(\vec{r},\beta),$$
(3)

where, $\beta = (\beta_2, \beta_4, \gamma)$ are deformation parameters. The pairing calculations are done within the BCS method including the Lipkin-Nogami method. E_{ω} represents the total routhian energy which is minimized at a fixed deformation (β) and angular frequency (ω) to get TRS. As the extension of these calculations in fixed angular momentum is straight forward as in Ref. [1]. After choosing the deformation parameters through configurationconstrained TRS, the aligned angular momenta are theoretically calculated. The dynamic moment of inertia (MoI), $\mathcal{J}^{(2)}$, is also calculated from the CSM, given by [6]

$$\mathcal{J}^{(2)} = \frac{d}{d\omega} \left\langle \Psi \right| J_x \left| \Psi \right\rangle, \tag{4}$$

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where $|\Psi\rangle$ is the eigenstate of the CSM Hamiltonian.

Results and Discussion

The nucleus ¹⁷⁵Re is suitable for studying configuration-constrained TRS calculations in one-quasiparticle rotational bands.



FIG. 1: TRS for configuration f at $\hbar \omega = 0.2$ (left) and $\hbar \omega = 0.4$ (right), before and after the crossing frequency ($\hbar \omega_c = 0.37$ MeV), respectively.

The deformation parameters $\beta_2 \approx 0.29$, $\beta_4 \approx -0.008$, and $\gamma \approx 0^\circ$ are chosen for four onequasiproton bands in ¹⁷⁵Re based on the TRS calculations. TRS for band 1, before and after crossing frequency is shown in Fig. 1. It is noted that the deformations vary for different configurations. The details of theoretical crossing frequencies ($\hbar\omega_c$) and alignment gains (Δi_x) are provided in Table I.

TABLE I: Theoretical crossing frequencies and aligned angular momenta for bands 1, 2, 3, and 4. The theoretical configurations along with the Nilsson orbital configuration are assigned according to parity (π) and signature (α) given by (π , α) = (-, -1/2) : e, g; (-, +1/2) : f, h; (+, +1/2) : a, c; and (+, -1/2) : b, d.

Band	Configuration	$i_x(\hbar)$	$\hbar\omega_c$	Δi_x
			(MeV)	(\hbar)
1	1/2 [541] (f)	3.27	0.37	8.6
2	1/2 [660] (a)	6.15	0.32	5.58
3	7/2 [404] (b)	1.89	0.34	10.31
4	5/2 [402] (c)	3.34	0.18	5.49

The band 1 is a negative parity band with configuration f. The bands 2, 3, and 4 are positive parity bands with configurations a, b, and c, respectively. These observations match well with the experimental observations [7].

The band crossings observed by sharp upbending are also evident in the dynamic MoI shown in Fig. 2 for bands 1, 2, 3, and 4.



FIG. 2: Theoretical dynamical MoI, $\mathcal{J}^{(2)}$, for bands 1, 2, 3, and 4.

The alignments and dynamic MoI for the various bands of 171,173 Re will be presented in the future. It will be interesting to observe how the two-quasineutron configurations influence the configuration assignment of the bands in 171,173,175 Re.

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