

# Shell-model study for nuclear structure properties of Pb isotopes

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## Introduction

The shell-model (SM) has proven to be an effective method for study and description of multi-particle systems, which helps to explain the various structures of nuclei on the Segré chart. Studying nuclei around  $^{208}\text{Pb}$  is significant both experimentally and theoretically, as this region offers an opportunity to observe and analyze complex structures and shapes. For instance, the unique shape staggering discovered in the mercury isotopes [1] that describes the coexistence of single-particle and collective degrees of freedom, the shape coexistence explored by  $\alpha$ -decay observed in nuclei around Pb [2] etc. To understand the nuclear structure, numerous experiments are being done at RIKEN, GSI/FAIR, and CERN to measure energy levels and electromagnetic properties in the Pb region. In this regard, we have studied Bi isotopes earlier [3]. Now, we aim to study systematic large-scale shell-model calculation for Pb isotopes by taking  $^{208}\text{Pb}$  as a doubly magic core. We have computed energies and electromagnetic characteristics and compared them with the existing experimental data. From the comparison with experimental data, we can also predict spins and parities of a number of unconfirmed states. We shall also talk about the calculated half-lives utilizing B(E2) values and the seniority of isomeric states.

## Theoretical Framework

The shell model Hamiltonian is written numerically in terms of single-particle energies and two-body matrix elements,

$$H = \sum_{\alpha} \varepsilon_{\alpha} \hat{N}_{\alpha} + \frac{1}{4} \sum_{\alpha\beta\delta\gamma JT} \langle j_{\alpha} j_{\beta} | V | j_{\gamma} j_{\delta} \rangle_{JT} A_{JT; j_{\alpha} j_{\beta}}^{\dagger} A_{JT; j_{\delta} j_{\gamma}},$$

where  $\alpha = \{nljt\}$  denote the single-particle orbitals and  $\varepsilon_{\alpha}$  is the corresponding single-particle energies.  $\hat{N}_{\alpha} = \sum_{j_z, t_z} a_{\alpha, j_z, t_z}^{\dagger} a_{\alpha, j_z, t_z}$  is the particle number operator.  $\langle j_{\alpha} j_{\beta} | V | j_{\gamma} j_{\delta} \rangle_{JT}$  are the two-body matrix elements coupled to good spin  $J$  and isospin  $T$ .  $A_{JT}^{\dagger}$  and  $A_{JT}$  are the fermion pair creation and annihilation operators, respectively.

Shell-model calculations have been carried out using two interactions: KHH7B and KHM3Y. The 14 orbitals make up the model space of the KHH7B interaction [4]. The cross-shell two-body matrix elements (TBMEs) are obtained from the H7B G-matrix [5], and the Kuo-Herling interaction [6] is used to develop the neutron-proton TBMEs, as modified in [7]. The second interaction employed here is KHM3Y; its model space have 24 orbitals. The M3Y interaction [8] provides the basis for the cross-shell two-body matrix elements for the KHM3Y interaction, whereas the neutron-proton interactions are derived from the Kuo-Herling interaction [6] as modified in Ref. [7]. To make calculations feasible, we have applied appropriate truncation. For KHH7B model space, we have completely filled proton orbitals below  $Z = 82$ , and there is no valence proton available to occupy orbital above  $Z = 82$ . For valance neutrons, we have allowed them to occupy in orbitals below  $N = 126$ , and allowed only those valance neutrons to occupy orbitals  $1g_{9/2}$ ,  $0i_{11/2}$ , and  $0j_{15/2}$ , which are

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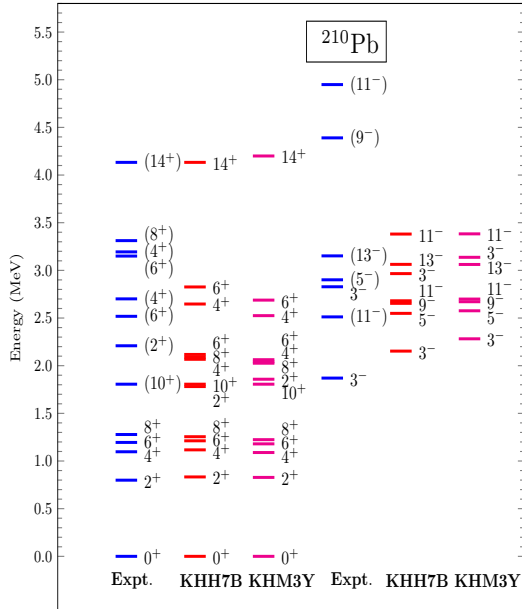


FIG. 1: Comparison between calculated and experimental energy levels [9] for  $^{210}\text{Pb}$ .

above 126. For KHM3Y model space, we have completely filled all the five proton orbitals below  $Z = 82$ , and there is no valance proton to occupy orbitals above  $Z = 82$ . The valance neutrons are allowed to fill in the orbitals below  $N = 126$ , and allowed only those neutrons to occupy in seven neutron orbitals ( $1g_{9/2}$ ,  $0i_{11/2}$ ,  $0j_{15/2}$ ,  $2d_{5/2}$ ,  $3s_{1/2}$ ,  $1g_{7/2}$ , and  $2d_{3/2}$ ), which are above  $N=126$ . Here, we have displayed result of  $^{210}\text{Pb}$  isotope only.

## Results and Discussion

The results of our calculations for the  $^{210}\text{Pb}$  isotope are shown in comparison with the experimental data in Fig. 1. Here, we have taken experimental energy states up to 5.0 MeV excitation energy. Only the yrast and non-yrast shell-model states corresponding to the available experimental data are presented here. All the positive as well as negative parity states above 1.8 MeV excitation energy are tentative except  $3^-$  states. Experimental states up to 1.806 MeV is reproduced very well by both the interactions. Experimentally, the lowest lying negative parity states is  $3^-$ , which

is reproduced by both the interactions nicely. Experimentally, ( $14^+$ ) lying at 4.133 MeV is supported by our SM calculations for  $14_1^+$  obtained at 4.133 and 4.200 MeV using KHH7B and KHM3Y interactions, respectively. Isomeric states in this isotope are  $6^+$  [ $\nu(g_{9/2}^2)$ ], and  $8^+$  [ $\nu(g_{9/2}^2)$ ]. Seniority ( $\nu$ ) of these states are 2 i.e. these states are formed due to one neutron pair breaking in  $g_{9/2}$  orbital. The calculated half-lives of  $6^+$ , and  $8_1^+$  isomers using B(E2) transition and internal conversion coefficient are 262 and 902 ns, respectively (corresponding experimental values are 92(10), and 166(15) ns). The calculated magnetic moment for  $6_1^+$ , and  $8_1^+$  are  $-2.542$  and  $-3.391 \mu_N$  which is close to their experimental value i.e.  $-1.872(90)$ , and  $-2.496(64) \mu_N$ , respectively. We have calculated the quadrupole moment for the  $6_1^+$  and  $8_1^+$  states, which are  $-0.042$ , and  $-0.22$  eb. We have taken standard values of effective charges i.e.  $e_p = 1.5e$ ,  $e_n = 0.5e$  and  $g_s^{free} = g_s^{eff}$ .

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## References

- [1] B. A. Marsh *et al.*, Nature Physics **14**, 1163 (2018).
- [2] A. N. Andreyev *et al.*, Nature **405**, 430–433 (2000).
- [3] S. Shukla, P. C. Srivastava and D. Patel, J. Phys. G: Nucl. Part. Phys. **51**, 075103 (2024).
- [4] N. A. F. M. Poppelier and P. W. M. Glaudemans, Z. Phys. A **329**, 275 (1988).
- [5] A. Hosaka, K.-I. Kubo, H. Toki, Nucl. Phys. A **444**, 76 (1985).
- [6] T. T. S. Kuo and G. Herling, US Naval Research Laboratory Report no. 2258 (1971) unpublished.
- [7] E.K. Warburton, B.A. Brown, Phys. Rev. C **43**, 602 (1991).
- [8] G. Bertsch, *et al.*, Nucl. Phys. A **284**, 39 (1977).
- [9] <http://www.nndc.bnl.gov/ensdf/>.