Probing deformation in transitional nuclei through N/Z parameterization

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Introduction

The role of inter-nucleon interactions in quantum shape transitions (QST) within stable nuclei has been extensively discussed (see [1-4]). A recent study [5] focused on first-order phase transitions in three mass regions, specifically for N= 40, 60 and 90. The nuclear systematics in these regions were parameterized against the neutron number (N), showing that observables within a shape transition region align with a singular seamless curve, with curves across different regions displaying similar structures.

Figure 1 (adapted from [5-8]) illustrate these trends for transitional nuclei at A~150, demonstrating significant similarity when parameterized against N.

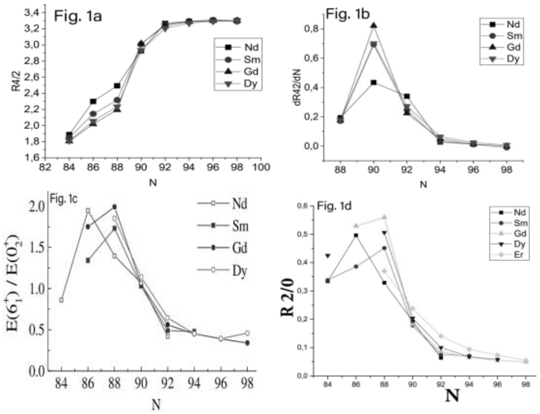

Fig.1: Universal trends in first order shape transitions for nuclei at A~150, plotted against N. Fig. 1a presents the Casten curves (adapted from [6]). Fig's. 1(b-d) depict observables [$dE_{4/2}/dN$, $E6_1^+/E0_2^+$ and $E_{2/0}$], aligning with smooth curves and consistent structural behavior.

The current article aims to explore parameterization of $dE_{4/2}/dN$ against the neutron-to-proton ratio (N/Z) instead of N as discussed in [5]. A virtually universal explanation of collectivity in heavy nuclei is obtained by using N/Z to parameterize the data instead of N, yielding a quantity with a more straightforward physical interpretation.

Method and calculations

We utilized the relationships

$$E_{4/2} = E_{41}{}^{+}/E_{21}{}^{+}$$

$$E_{2/0}= E_{21}{}^{+}/E_{02}{}^{+}$$

$$E(6_1{}^{+})/E(0_2{}^{+}) \text{ and}$$

$$dE_{4/2}/dN= E_{4/2}(N)-E_{4/2}(N-2).$$

see Figs.1 and 2 for details.

Table 1: Table showing the energy ratios $E_{4/2}$ and their rate of change, $dE_{4/2}/dN$ (N), as functions of the neutron number N and N/Z values for even- even nuclei with N = 90 located near the maxima illustrated in Fig. 2.

Nuclei	N/Z	$E_{4/2}$	$dE_{4/2}/dN$
Ce	1.55	2.86	0.28
Nd	1.50	2.92	0.43
Sm	1.45	3.00	0.69
Gd	1.40	3.01	0.82
Dy	1.36	2.93	0.70
Er	1.32	2.74	0.43

Result and discussion

Figure 2 shows the energy systematics of $dE_{4/2}/dN$ plotted against the N/Z ratio for six transitional nuclei in the A~150 mass region. The N/Z ratio, which is closely related to nuclear binding energy and stability, serves as a useful

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*Electronic address:*mudasir.42100330@lpu.in* indicator of nuclear structure. Stable nuclei typically have specific N/Z ratios—around 1 for light nuclei and increasing to about 1.6 for heavier ones.

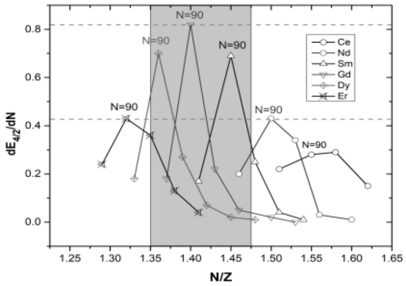

Fig. 2 Energy systematics of $dE_{4/2}/dN$ versus N/Z ratio for six transitional nuclei in the A~150 mass region, highlighting a distinct kink at N=90 that diminishes as N/Z decreases, with a transition to deformed structure for N>90 in Sm-Dy nuclei.

A more careful examination of Fig. 2 reveals a detailed and physically meaningful structure, specifically a noticeable kink in the curves at N=90 for all nuclei (Ce-Er), which becomes progressively less distinct as N/Z decreases. This kink is immediately apparent from the plots. This observation aligns with previous studies [5-8], which noted kinks at N=90 (for Nd-Dy) and at N=86 (for Nd) and N=88 (for Sm-Er) (refer to Fig. 1). Additionally, the curves display a significant spread—an otherwise hidden feature when plotted against N alone [5-8], where the coalescence of curves obscured this aspect. This spread helps in understanding shell evolution beyond magic nuclei as N/Z changes (e.g., increasing N/Z suggests structural stability). Moreover, the curves for Nd and Sm signify that, before the upward trend at N=90, the curves asymptotically approach an N/Z value of ~1.5, which happens to be the threshhold for ɣ-unstable nuclei and close to the value for triaxial nuclides with a substantial gamma. This contradicts previous conceptions but is reasonable, given the current analysis, which shows that the Z=64 shell closure operates efficiently for N=90 but disappears for N>90. For N=90, Gd exhibit $dE_{4/2}/dN$ values around 0.8 (a peak value—refer to top horizontal line in Fig. 5), and nuclei like Nd and Sm with proton holes (Z=50−64 shell greater than partially-filled) and neutron particles are are expected to demonstrate a proclivity for triaxiality. For N>90, the influence of the Z=64 closed shell disappears, and nuclides from Sm-Dy (refer to the shaded vertical section in Fig.2) transit to a particle-particle character (Z=50−82 shell less than partially occupied), developing a distorted structure, as indicated by the suddenly reduced $dE_{4/2}/dN$ values (~0.4) and N/Z values receding 1.50. Lastly, the tendency toward triaxiality for Nd and Sm nuclei with N=90 is corroborated by Casten *et al.* [6], who demonstrated that the quasi-gamma-band energy reaches its minimum in this area. These findings offer a deeper understanding of nuclear shape deformation and can be extended to other mass ranges.

Conclusion

In conclusion, our analysis reveals that while the regions examined display similar trends in the energy ratios $E_{4/2}$ and $E_{2/0}$, with notable maxima occurring around neutron number N = 88 and N = 90, the distinct feature of our findings is the observation of a consistent maximum at N = 90 across all nuclei when plotted against the N/Z ratio. This uniformity highlights a unique characteristic of the collective neutron number, marking N = 90 as a significant point of interest in the study of nuclear collectivity transitions.

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