Study of 2^+ Resonance in $^4He(d,d)^4He$ using Reference Potential Approach

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Introduction

Scattering studies between light nuclei is the only way by which one can understand the level structure of the nucleus with small Z values. Analyzing $^4He(d,d)^4He$ elastic scattering data by considering energy-dependent phase shifts gives rise to the energy levels of ^6Li nuclei. Deuteron-alpha (dα) scattering is also of interest in connection with the problem of measuring the spin polarization parameters characterizing deuteron beams. Over the past few decades, $^4He(d,d)^4He$ scattering has been extensively investigated for the low-lying levels of ^6Li, both experimentally and theoretically [1, 2]. In this study, we determine the underlying interaction potential for the resonance states of $^3D_1$ and $^3D_2$ of the dα system by utlizing the reference potential approach[3]. The obtained scattering phase shifts from the underlying potentials are utilized to calculate the respective partial cross-sections and resonance energies.

Methodology

The phase equation for $^3D_1$ and $^3D_2$, with $\ell = 2$, is given by

$$\delta_2'(k,r) = -\frac{U(r)}{k}[-\sin(\delta_2 + kr) - \frac{3\cos(\delta_2 + kr)}{kr} + \frac{3\sin(\delta_2 + kr)}{kr^2}]^2 \quad (1)$$

where one needs to provide a mathematical function to represent the form for the potential U(r). This will be constructed as a combination of three Morse functions over three different regions of interaction, with the third Morse function being negative to account for the Coulomb interaction [3], connected in a piece-wise smooth manner. Each Morse function comprises of 4 parameters which results in 12 parameters in addition to two boundary points. By applying the continuity conditions for the three Morse functions and their derivatives at the two boundaries, one can solve the resulting four equations and obtains 4 of the parameters to be dependent on other parameters. Hence, the total number of the model parameters is reduced to 10. Varying the two boundary points further provides flexibility and hence greater variety. The 10 parameter reference potential is given as input to solve the phase equation, (eqn. ref1), numerically using RK-5 method with the initial condition as $\delta_2(k,0) = 0$. The final integration distance can be chosen as far as possible. The phase equation is solved by updating the model parameters in each iteration, using a genetic algorithm[3], so as to minimize mean absolute percentage error (MAPE) between the obtained and expected scattering phase shifts.

Results and Discussion

Varying the parameters of the reference function will produce a wide range of curves in the sample space. While, the inverse potential that best matches the expected phase shifts is determined by optimizing the reference function's parameters to minimize a cost function.

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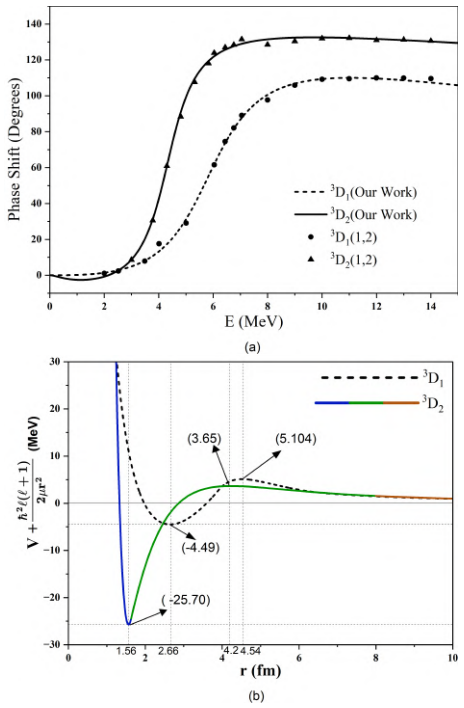

FIG. 1: (a) Phase shifts and (b) Interction potentials for $^3D_1$ and $^3D_2$ states

For low-energy data in the range of 3–6 MeV, phase shifts have been obtained from ref.[2]. However, to accurately capture the resonance peak, phase shifts up to 15 MeV, as provided in ref.[1], need to be considered Both the obtained and expected phase shifts are plotted in FIG.1(a). The corresponding potentials for the two states are shown in FIG.1(b), by adding the centrifugal term for $\ell = 2$. For $^3D_1$, we have obtained the potential depth, V_D to be 4.49 MeV and Coulomb barrier height, V_{CB} to be 5.104 MeV with MAPE to be 0.10. Similarly, for $^3D_2$, we obtained V_D = -25.70 MeV and V_{CB} = 3.65 MeV having MAPE of 0.017. It is important to note that we have not considered any Coulomb ansatz explicitly in the choice of our reference potential, except utilizing an inverted Morse function in the third region of interaction. Using the obtained scattering phase shifts, the partial cross sections for $^3D_1$ and $^3D_2$ are obtained at different lab energies and are shown in FIG.2. For $^3D_1$,

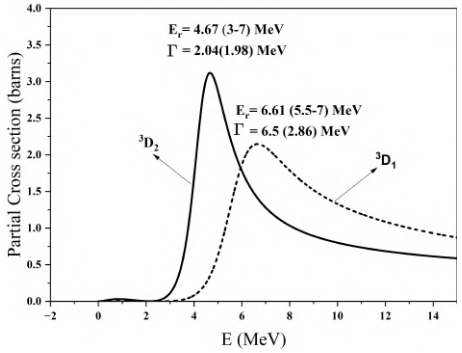

FIG. 2: Partial cross section for resonance states

the resonance energy is obtained at $E_r = 6.61$ MeV while experimental value is in the range of 5.5 to 7 MeV. It's width is 6.5 MeV and reflects a broad resonance as seen in Fig. 2. The quoted value is 2.86 MeV which is not in tune with what one would expect, either from the nature of the phase shifts or from the partial cross section curve. For $^3D_2$, the computed resonance energy is $E_r = 4.67$ MeV while the experimental value is in the range of 3 to 7 MeV. It's width is 2.04 MeV, while the experimental value is 1.98 MeV. These will get further validated when the total cross sections are obtained by considering the partial cross sections due to other partial waves.

To conclude, we have constructed the underlying interaction potentials for the $^3D_1$ and $^3D_2$ states of dα system by optimizing the reference potential parameters and the obtained resonances are well within the expected range from experiments.

References

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