Calculation of Reaction Cross-Sections for Secondary Particle Production in Proton Beam Therapy Using Glauber Model

Aishwarya G.[1], Tamil Selvan S[1], P. Raj Preethi[1], Santosh K. Agarwalla[2], Jajati K. Nayak[3], Nrusingh C. Biswal[4] P. Prema[1]

[1]*Department of Physics, Amrita School of Physical Sciences Coimbatore, Amrita Vishwa Vidyapeetham, India.*
[2]*Department of Physics, Fakir Mohan University, Balasore - 756019, India.*
[3]*Variable Energy Cyclotron Centre, Kolkata - 700064, India.*
[4]*Department of Radiation Oncology, University of Maryland School of Medicine, Baltimore, MD 21201, USA*

Introduction:

Proton beam therapy is a form of radiation therapy that targets cancer cells using ionizing radiation. Its key advantage over photon therapy is the Bragg peak, which allows precise tumor targeting while sparing surrounding healthy tissues. However, the steep dose falloff after the Bragg peak introduces range uncertainties, necessitating careful monitoring of dose distribution during treatment.

Understanding proton interactions with matter is crucial to determining how dose is deposited within the body. These interactions inelastic Coulomb scattering, elastic scattering, and inelastic nuclear reactions generate secondary particles like neutrons, positrons, alpha particles, and gamma rays. Secondary particles are vital for verifying the proton beam's range and can contribute to the dose delivered to tissues surrounding the tumor.

This study aims to calculate the total number of secondary particles produced during the interaction of protons with key elements in human tissues, such as carbon, nitrogen, and oxygen, using the Glauber model, thereby providing essential data to account for additional dose deposited by secondary particles in treatment planning.

Glauber Model:

Glauber theory provides a microscopic framework for describing the scattering of composite particles. It models nucleus-nucleus interactions by considering nucleon-nucleon interactions, as shown in Figure 1, assuming that the nucleus follows a straight-line trajectory.

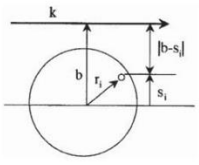

Figure 1: Nucleon-nucleus scattering [3].

The proton-nucleus reaction cross-section is expressed as

$$\sigma_R = \int d\boldsymbol{b}(1 - |e^{i\chi(\boldsymbol{b})}|^2)$$

The of proton-nucleus phase shift function in Optical Limit Approximation (OLA) is expressed as

$$e^{i\chi(\boldsymbol{b})} = \exp\left[i\chi_{pn}(\boldsymbol{b}) + i\chi_{pp}(\boldsymbol{b})\right]$$

where,

$$i\chi_{pn}(\boldsymbol{b}) = -\int d\boldsymbol{r}\rho_n(\boldsymbol{r})\Gamma_{pn}(\boldsymbol{b}-\boldsymbol{s})$$

$$i\chi_{pp}(\boldsymbol{b}) = -\int d\boldsymbol{r}\rho_p(\boldsymbol{r})\Gamma_{pp}(\boldsymbol{b}-\boldsymbol{s})$$

The profile function is given by

$$\Gamma_{pN}(b) = \frac{1 - i\alpha_{pN}}{4\pi\beta_{pN}^2}\sigma_{pN}^{tot} exp\left(-\frac{b^2}{2\beta_{pN}^2}\right)$$

where σ_{pN}^{tot} is the total proton-nucleon cross-section. β_{pN} and α_{pN} are fitting parameters determined from the elastic scattering differential cross sections. The density distribution of the protons and neutrons is given by the two-parameter Fermi density distribution function.

Coulomb Modified Glauber Model:

The Glauber model works well for high-energy interactions but needs adjustments at lower energies. When the center-of-mass energy drops below 30 times the Coulomb barrier, Coulomb interactions cause significant deflection from straight line trajectory [1]. Corrections are applied by multiplying the reaction cross-section with a Coulomb correction factor

$$\sigma_R' = \sigma_R\left[\frac{1 - z_p z_t e^2}{E_{cm}R_{eff}}\right]$$

where E_{cm} is the energy in centre of mass frame and R_{eff} denotes the effective interaction radius.

Secondary particle Flux:

Flux of secondary particles produced per second is given by

$$\phi = n_T I \int \frac{\sigma(E)}{dE/dx} dE$$

where,

n_T is the number of target nucleus per unit volume.

I is the incident proton flux per second.

σ(E) is the reaction cross-section.

dE/dx is the stopping power.

Results and discussion:

The reaction cross-section was determined using the optical limit approximation (OLA) and the Coulomb-modified Glauber model. Table 1 compares our calculated cross-sections with those from [2]. Below 200 MeV, the OLA values closely match the TALYS code results. However, above 200 MeV, as shown in Figure 2, a deviation between the TALYS and OLA values is observed.

Table 1: Comparison of reaction cross-section values to the data in [2].

E	σ_{expt}	σ_{theo}	%error
40	43.20	43.11	0.22
100	28.40	28.38	0.08
200	21.80	21.77	0.15
300	20.20	20.15	0.23
425	20.00	19.97	0.17
550	21.70	21.65	0.22
650	23.30	23.26	0.17
800	24.30	24.26	0.17

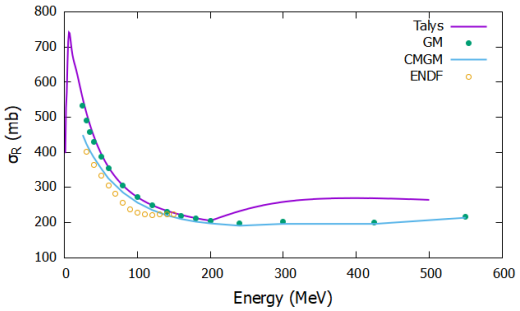

Figure 2: Reaction cross-section for p + ^{12}C

A similar calculation for p + ^{14}N and p + ^{16}O reactions is presented in Figure 3.

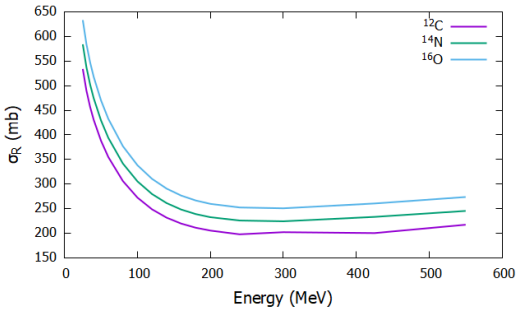

Figure 3: Comparison of reaction cross-section of proton beam interaction with different nuclei.

The flux of secondary particles produced by proton interactions with various tissues, as shown in Figure 4, is calculated using the reaction cross-section, stopping power, and tissue composition data from Table 2.

Table 2: Density and composition of various tissues [4].

Tissue Type	Density	C	N	O
Adipose tissue	0.95	59.8	0.7	27.8
Brain	1.04	14.5	2.2	71.2
Muscle	1.05	14.3	3.4	71.0
Breast tissue	1.02	33.2	3.0	52.7
Blood	1.06	11.0	3.3	74.5

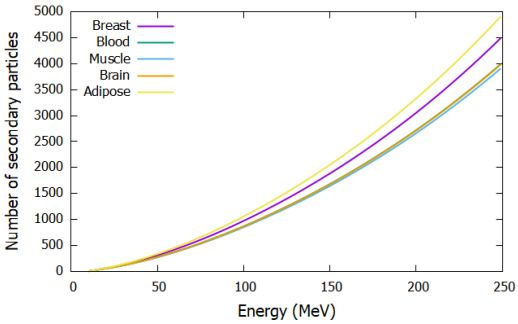

Figure 4: Number of secondary particles produced from different tissues.

Conclusion:

The reaction cross-sections obtained using OLA agree with TALYS, validating our approach. The Glauber model bridges the gap between theoretical and experimental results, reducing the need for simulations.

The secondary charged particle flux, calculated from reaction cross-sections and stopping power, aids in optimizing cancer treatment. As these secondary particles contribute to dose deposition, understanding their flux allows for precise adjustments in treatment planning, enhancing therapeutic accuracy.

References:

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