# Comparison of first- and full-order Coulomb breakup of exotic nuclei

H. Kushwaha* and R. Chatterjee
*Department of physics, Indian Institute of Technology Roorkee, Roorkee - 247667, INDIA*

## 1. Introduction

The Coulomb breakup is a crucial technique for investigating halo nuclei. However, it remains a complex process involving at least three interacting bodies and the long-range Coulomb force. This complexity makes an exact treatment of the breakup process particularly challenging. This work presents a theoretical framework that attempts a perturbative expansion of the breakup amplitude within the framework of the post-form Finite Range Distorted Wave Born Approximation (FRDWBA). Accurately incorporating finite range effects within this approach is intricate, necessitating the evaluation of six-dimensional integrals with asymptotically oscillatory functions. To tackle this problem, we use an approximation known as the "local momentum approximation (LMA)," which aids us in studying the Coulomb breakup of one-neutron exotic nuclei in the light and deformed medium mass region [1].

An intriguing aspect emerges when the transition amplitude is expanded to the first order, as explored in Refs. [2, 3]. We extend the core concept of these studies but introduce a modified approach. Specifically, we incorporate momentum-space Coulomb wave functions into the dynamic part of our calculations and then compare our findings with those derived from the full-order theory.

This contribution will primarily focus on calculating inclusive reaction observables like the relative energy spectra with our first-order method.

## 2. Methodology

We begin our discussion by considering the breakup reaction $a + t \rightarrow b + n + t$. In this case, the projectile $a$ breaks into core $b$ and neutron $n$ in the Coulomb field of the target $t$. We utilize the post-form T-matrix within the FRDWBA framework to study this reaction. The reduced transition amplitude $\beta_{lm}$, for the process is

$$\begin{aligned}\hat{l}\beta_{lm}(\mathbf{q}_b,\mathbf{q}_n;\mathbf{q}_a) &= \int\int d\mathbf{r}_1 d\mathbf{r}_i \chi_{q_b}^{(-)*}(\mathbf{r}_b) \\ &\times \chi_{q_n}^{(-)*}(\mathbf{q}_n,\mathbf{r}_n)V_{bn}(\mathbf{r}_1)\phi_a(\mathbf{r}_1)\chi_{q_a}^{(+)}(\mathbf{r}_i),\end{aligned} \tag{1}$$

where, $\hat{l} = \sqrt{2l+1}$ and $l$ as the relative orbital angular momentum of the $b-n$ system. $V_{bn}(\mathbf{r}_1)$ is the interaction between $b$ and $n$, in the initial channel. $\chi_{q_b}$ and $\chi_{q_n}$ are the distorted waves for fragment $b$ and $n$. $\chi_{q_a}$ is a distorted wave for projectile $a$, and $\phi$'s are concerned particles' internal state wave functions. $\mathbf{q}$'s are Jacobi wave vectors for respective particles. As fragment $n$ is a neutron, it will not have electromagnetic interactions with the target; hence, $\chi_{q_n}$ is taken as a plane wave.

Solving Eq. (1) poses a significant challenge, as it necessitates the evaluation of a six-dimensional integral. By employing an effective LMA [1], the transition amplitude $\beta_{lm}$ can be factorized into two components: the structural component $Z_{lm}$ [1, 3], and the dynamic component $I_d$,

$$I_d = \int d\mathbf{r}_i \chi_b^{(+)}(-\mathbf{q}_b,\mathbf{r}_i)e^{-i\lambda\mathbf{q}_n\mathbf{r}_i}\chi_a^{(+)}(\mathbf{q}_a,\mathbf{r}_i), \tag{2}$$

where, $\lambda = m_t/(m_b+m_t)$. The distinctive aspect of this work lies in utilizing the momentum-space representation of Coulomb distorted wave functions to evaluate $I_d$. With,

$$\chi_{q_j}^{(+)}(\mathbf{r}_i) = \frac{1}{(2\pi)^{3/2}}\int d^3\mathbf{Q}e^{i\mathbf{Q}.\mathbf{r}_i}\tilde{\chi}_{q_j}^{(+)}(\mathbf{Q}), \tag{3}$$

the dynamics part $I_d$, up to first order in $\alpha$, simplifies to

$$\begin{aligned}I_d \approx \frac{8\pi m_a c Z_t Z_b \alpha}{\hbar(\mathbf{q}_a-\mathbf{q}_b-\delta\mathbf{q}_n)^2}&\left[\frac{1}{[\mathbf{q}_a^2-(\mathbf{q}_b+\delta\mathbf{q}_n)^2]}\right. \\ &\left.+\frac{m_b}{m_a}\frac{1}{[\mathbf{q}_b^2-(\mathbf{q}_a-\delta\mathbf{q}_n)^2]}\right],\end{aligned} \tag{4}$$

---

[*] Electronic address: `harsh_k@ph.iitr.ac.in`

where $\alpha$ is the fine structure constant, $Z_t$ and $Z_b$ are the atomic numbers of the target $t$ and $b$. $m_b$, and $m_a$ are the masses of fragment $b$ and projectile $a$, and $c$ is the speed of light in vacuum. We will call our approach the "Fourier Transformation Method".

One can then calculate various inclusive reaction observables like the angular distribution of the neutron, relative energy, and momentum distributions directly from the triple differential cross sections with appropriate Jacobians and proper three-body kinematics.

## 3. Results and Discussion

We utilized our method to determine relative energy spectra ($d\sigma/dE_{bn}$) for the elastic Coulomb breakup of the halo nucleus $^{11}$Be into neutron and core $^{10}$Be, in the presence of heavy target $^{208}$Pb, at various incident beam energies (E$_a$). It has been shown that the peak position of the relative energy spectra is correlated with the binding energy of the halo system [4].

Furthermore, the ground state wave function of the projectile $^{11}$Be [$^{10}\text{Be}(0^+) \otimes 2s_{1/2}\nu$], is assumed to be derived from a phenomenological Woods-Saxon potential whose depth is adjusted to reproduce the one-neutron separation energy of 0.504 MeV.

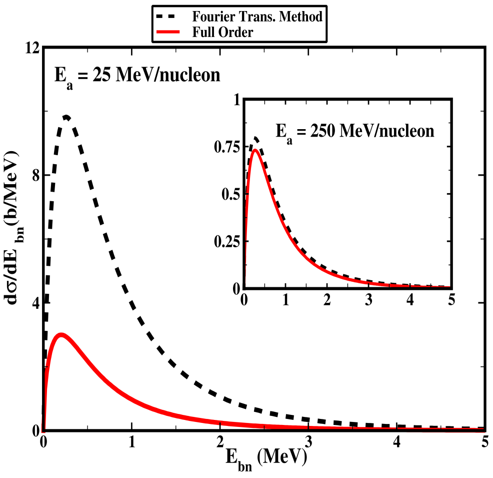

FIG. 1: Relative energy spectra for the Coulomb breakup of $^{11}$Be on $^{208}$Pb.

In Fig. 1, we present our preliminary results at two incident beam energies $E_a = 25$ MeV/nucleon and 250 MeV/nucleon (inset) for the relative energy spectra (dashed line) computed with the Fourier Transformation Method. They are also compared with those (solid line) from the full-order theory [1].

The centre of mass projectile-target angle $\theta_{at}$ is integrated from 0° to 40°. The corresponding azimutal angle $\phi_{at}$ and the $b-n$ solid angles $\Omega_{bn}$ are integrated to kinematically allowed values for both incident beam energies.

We note that the results obtained significantly differ at lower beam energy (E$_a$ = 25 MeV/nucleon). This discrepancy is primarily due to the significant influence of higher-order effects, e.g. postacceleration, which are not accounted for in the first-order method. However, both theories yield similar results at higher beam energy (E$_a$ = 250 MeV/nucleon) because of the negligible impact of higher-order effects. These findings are consistent with the conclusions presented in Ref. [3].

Thus, at higher beam energies in the range of 250-350 MeV/nucleon or above, as is the norm in several accelerator facilities worldwide, our perturbative method would be quite suitable. Additionally, extending our method to proton halo breakup would be straightforward, which is still very difficult in the full-order theory. Coupled with the fact that the associated numerics in our method are relatively simpler than the full-order method, it would also be interesting to attempt calculations of radiative capture cross-sections with our theory.

We also plan to present other inclusive observables with our method.

## 4. Acknowledgement

The Indo-Japan Cooperative Science Programme (IJCSP)-2023 for DST-JSPS grant TPN-98540. The MoE, India, for the SPARC/2018-2019/P309/SL grant and CSIR for research fellowships to [HK].

## References

[1] R. Chatterjee and R. Shyam, Prog. Part. Nucl. Phys. **103** (2018) 67.

[2] G. Baur, K. Hencken, and D. Trautmann, nucl-th/0108013.

[3] P. Banerjee, G. Baur, K. Hencken, R. Shyam and D. Trautmann, Phys. Rev. C **65** (2002) 064602.

[4] R. Chatterjee, L. Fortunato, and A. Vitturi, Eur. Phys. J. A **35** (2008) 213.