Exploring ground state deformation of ^{28}Si via fusion barrier distribution: A Bayesian approach

Chandra Kumar and S. Nath*
Inter-University Accelerator Centre, Aruna Asaf Ali Marg, New Delhi 110067, India

In recent decades, significant efforts have been made to understand the transition of nuclear shapes from spherical to deformed configurations through both low-energy [1–4] and high-energy collisions [5]. In low-energy domain, fusion barrier distribution ($\mathcal{D}$) is crucial for revealing detailed information about the structure of collision partners. The shape of $\mathcal{D}$ had been observed to be highly sensitive to nuclear structure (see *e.g.*, [1]). $\mathcal{D}$s are extracted directly from the fusion excitation functions, as demonstrated by Rowley *et al.* [6], using the "double differential" (DD) method. The results suffer from several shortcomings, including ambiguity problem related to the energy step size used in the differentiation process and large uncertainties at higher energy regions. As an alternative to the *DD* recipe in fusion studies, extensive research has been conducted on $\mathcal{D}$ extracted from quasielastic (QEL) measurements [7]. While this first-derivative approach reduces uncertainties in higher energy regions, it encounters poor sensitivity to low peaks at these energies [7]. A recent study by Jiang *et al.* [8] showed that an analytic approach, based on the multi-Gaussian barrier height distribution, provides enhanced resolution across all energy regions. Result obtained from this method is largely unaffected by the energy step size and also remains robust against uncertainties in the measured cross sections, thus providing new opportunities to investigate nuclear structure via fusion dynamics. Consequently, we focus on ^{28}Si, a nucleus whose classification as either rotational or vibrational remains a topic of considerable debate in the literature. Recently, Gupta *et al.* [4] suggested that ^{28}Si behaves as a rotational nucleus. The authors found that the barrier distribution derived from QEL scattering was significantly affected by the sign of β_2 and that the choice of prolate or oblate shape for ^{28}Si dramatically influenced the results, implying that ^{28}Si might be oblate rather than prolate in shape. Despite many fusion measurements indicating significant hexadecapole deformation in ^{28}Si [9], a very small value of β_4 was reported in Ref. [4]. Further exploration and evaluation of the robustness of this procedure are reported in the present work. Specifically, the fusion of ^{28}Si with the spherical target ^{144}Sm [10] is analyzed within the coupled-channels (CC) framework to extract β_2 and β_4 of ^{28}Si.

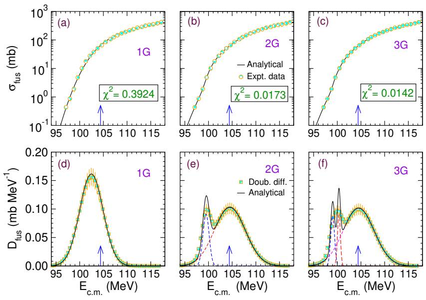

FIG. 1: (a-c) Excitation functions and (d-f) barrier height distributions for ^{28}Si+^{144}Sm, derived from 1G, 2G and 3G analytic methods. Individual Gaussians in 2G and 3G fits are also shown in panels (e) and (f).

We used the analytic method [8] with up to three-Gaussian (3G) least-squares fit to extract the $\mathcal{D}$ for the present system. The resulting excitation functions and $\mathcal{D}$s are shown in Fig. 1. CC calculations for fusion were performed using a modified version of CCFULL [11]. Two different sets of calculations were performed, considering ^{28}Si either to be rotational or vibrational in nature. For rotational coupling, we explored a broad param-

---

[*] Electronic address: `subir@iuac.res.in`

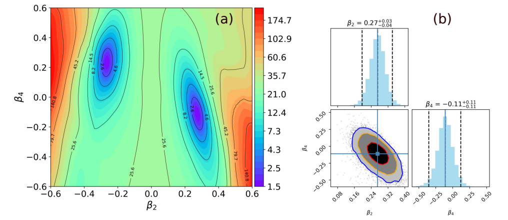

FIG. 2: (a) χ^2 distribution as a function of β_2 and β_4 and (b) the posterior distributions for prolate shape of ^{28}Si with 95% confidence interval.

eter space for β_2 and β_4 of ^{28}Si, both ranging from -0.60 to $+0.60$, with a step size of 0.01. In contrast, for the vibrational coupling, only β_2 was varied. In both cases, the couplings of ^{144}Sm was fixed, with one-phonon excitation for both the quadrupole (2^+) and the octupole (3^-) modes, including all mutual excitations. The least-squares fit was performed between the analytic (3G) and theoretical $\mathcal{D}$s. The resulting χ^2 distributions in the two-dimensional space of β_2 and β_4 are shown in Fig. 2 (a). This figure reveals two distinct minima, one corresponding to an oblate shape (left contour) and the other to a prolate shape (right contour). Next, we performed a Bayesian analysis using a Markov-Chain Monte Carlo (MCMC) framework to quantify the deformation parameters β_2 and β_4, along with their uncertainties. Uniform priors were used for β_2 and β_4, with a Gaussian likelihood function based on the chi-squared statistic. The `emcee` Python package, implementing the Affine-Invariant MCMC Algorithm [12, 13], was employed to sample the posterior distribution. For the prolate shape, the mean values were $\beta_2 = 0.27^{+0.03}_{-0.04}$ and $\beta_4 = -0.11^{+0.11}_{-0.11}$, whereas for the oblate shape, the mean values were $\beta_2 = -0.26^{+0.03}_{-0.03}$ and $\beta_4 = +0.23^{+0.09}_{-0.10}$. The mean value of β_2 was found to be $0.28^{+0.06}_{-0.06}$ for the vibrational mode. The experimental fusion excitation functions and the $\mathcal{D}$s are compared with CC calculations in Fig. 3(a) and Fig. 3(b), respectively. The CC calculations, irrespective of whether ^{28}Si was treated as a rotational (prolate or oblate) or a vibrational nucleus, yielded similar fusion excitation functions. One may note that both the experimental and the theoretical $\mathcal{D}$s (shown in Fig. 3(b)) have two distinct peaks. A detailed examination of the theoretical $\mathcal{D}$s indicates that the peak at 104.5 MeV aligns better with results based on rotational couplings for ^{28}Si. However, the results are not sufficiently sensitive to differentiate between the two probable shapes of ^{28}Si or to precisely constrain β_4.

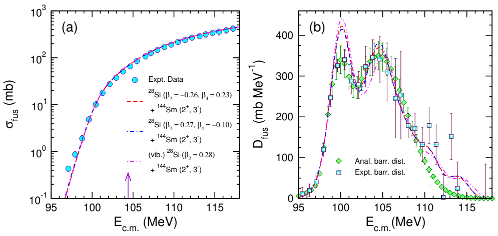

FIG. 3: Comparison of measured fusion (a) excitation function and (b) barrier distribution with CC calculations for ^{28}Si+^{144}Sm. A 10% constant uncertainty was assumed for the analytic $\mathcal{D}$.

This study highlights that results about deformation parameters, obtained from heavy ion-induced reactions, can be ambiguous and caution must be applied while drawing definitive conclusions from similar studies.

References

[1] R. C. Lemmon *et al.*, Phys. Lett. B **316**, 32 (1993).

[2] C. R. Morton *et al.*, Phys. Rev. C **64**, 034604 (2001).

[3] H. M. Jia *et al.*, Phys. Rev. C **90**, 031601(R) (2014).

[4] Y. K. Gupta *et al.*, Phys. Lett. B **845**, 138120 (2023).

[5] Wouter Ryssens *et al.*, Phys. Rev. Lett. **130**, 212302 (2023).

[6] N. Rowley *et al.*, Phys. Lett. B **254**, 25 (1991).

[7] H. Timmers *et al.*, Nucl. Phys. A **584**, 190 (1995).

[8] C. L. Jiang and B. P. Kay, Phys. Rev. C **105**, 064601 (2022).

[9] Gurpreet Kaur *et al.*, Phys. Rev. C **97**, 064606 (2018).

[10] M. Dasgupta *et al.*, Proc. Workshop on Heavy Ion Fusion, Padova, Italy, May 25-27 1994, p. 115.

[11] K. Hagino *et al.*, Comput. Phys. Commun. **123**, 143 (1999).

[12] J. Goodman *et al.*, Commun. Appl. Math. Comput. Sci. **5**, 65 (2010).

[13] D. Foreman-Mackey *et al.*, Publ. Astron. Soc. Pac. **125**, 306 (2013).