Two and Three Body Resonance in ^{12}C

Arun K. Jain[1,2*] and B. N. Joshi[1]
[1]Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai and
[2]Sri Satya Sai Institute of Higher Learning, Putthaparthy, Prasanthi Nilayam.

It is a puzzle to understand the occurance and nature of two 0^+ states of ^{12}C, the Hoyle state and the 9.93 MeV state (Fig1). We attempted to understand these in-terms of two $^8Be_{(g.s.)}$ resonances pertrubing each other in the ^{12}C nucleus. (see Fig.2)[1, 2]

There are three α- particles in ^{12}C to start with two $^8Be_{(g.s.)}$ resonances in the formation of ^{12}C. In this the 3-α- particles in ^{12}C at the corners of the triangle ABC have two $^8Be_{(g.s.)}$ resonances for the AC and BC sides[3, 4]. The hamiltonian, H for this 3-α particle state is,

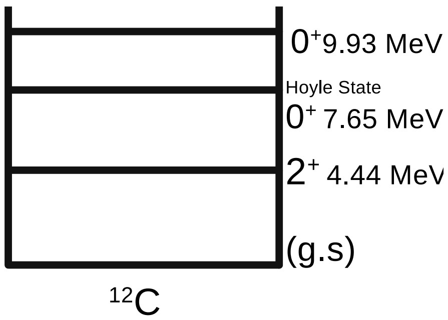

FIG. 1: First three excited state of ^{12}C.

$$H = T_A + T_B + T_C + V_{AB}(r_{AB}) + V_{AC}(r_{AC}) + V_{BC}(r_{BC}) = E_A + E_B + E_C$$

where T_i is the kinetic energy operator for the i^{th} particle and $V_{ij}(r_{ij})$ is the potential energy operator for the i and j α-particles in terms of relative coordinates, $\vec{r}_{ij}=\vec{r}_i - \vec{r}_j$. Thus $\vec{r}_{BC}=\vec{r}_B$-$\vec{r}_C$, $\vec{r}_{AC}=\vec{r}_A$-$\vec{r}_C$ and $\vec{r}_{AB}=\vec{r}_A$ - $\vec{r}_B$ such that $\vec{r}_{AB}=\vec{r}_{AC}$ - $\vec{r}_{BC}$. Now, we have the hamiltonian as,

$$H = T_{AC} + T_{BC} - \frac{\vec{p}_{AC} \cdot \vec{p}_{BC}}{m_c} + V_{AB}(r_{AB}) + V_{AC}(r_{AC}) + V_{BC}(r_{BC})$$

Here T_{ij} is the relative kinetic energy operator between particle i and j and $\vec{p}_{ij}$ is the momentum operator for particle i and j. In this equation it is seen that the $(\vec{p}_{AC} \cdot \vec{p}_{BC})/m_C$ term couples the relative motion of A, B and C besides the $V_{AB}(r_{AB})$ doing the same coupling. From Fig.3 we notice that of the $F_\parallel$ and $F_\perp$ components of $\vec{F}_{AC}$ (Force between A and C) and $\vec{F}_{BC}$ (Force between B and C) the $F_\parallel$ components cancel and $F_\perp$ component gives the DC motion. The hamiltonin H then becomes,

$$H = T_{AB} + T_{DC} + V'_{AB}(r_{AB}) + V^{'}(R) + E_{cm} = E_{AB} + E_{DC} + E_{cm}$$

Thus the coupling is removed and the Hamiltonian H is separable in $\vec{r}$ and $\vec{R}$ variables, where $\vec{r}=\vec{r}_{AB}$ and $\vec{R}=\vec{r}_{BC}=\vec{r}_B$-$\vec{r}_C$. A resonance in r variable will be two body resonance between A and B, while a resonance in R variable may be called a 3-body resonance. From the harmonic oscillator shell model we know that $^{12}C_{(g.s.)}$ has $8\hbar\omega$ while the $^8Be_{(g.s.)}$ has $4\hbar\omega$ quanta. Thus the $\vec{r}$ and $\vec{R}$ motions will have $4\hbar\omega_r$ $4\hbar\omega_R$ quanta each.

For the 4.43 MeV 2^+ state one has E_ℓ=4.43MeV= $\frac{\ell(\ell+1)\hbar^2}{2T_R}$= $\frac{6\hbar^2}{2 \times \frac{3}{2} m_\alpha R^2}$

Hence $R^2 \simeq 5$ fm^2 or R$\simeq$ 2.3 fm and R_{DC}=3/2R $\simeq$ 3.5 fm.

Hence $R_{AC} > 3.5$ fm / cos $\theta_R \gg 2R_\alpha \simeq$ 3.3 fm

Hence ABC is an isosceles triangle and not an equilateral triangle, where $R_{AB} < R_{AC}$

From these estimates one can conclude that $\hbar\omega_r > \hbar\omega_R$, because in an oscillator, angular frequency ω is inversely proportional to

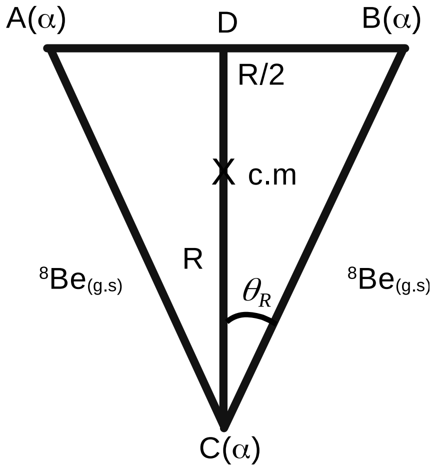

FIG. 2: The two $^8Be_{g.s}$ resonances in the formation of ^{12}C nucleus.

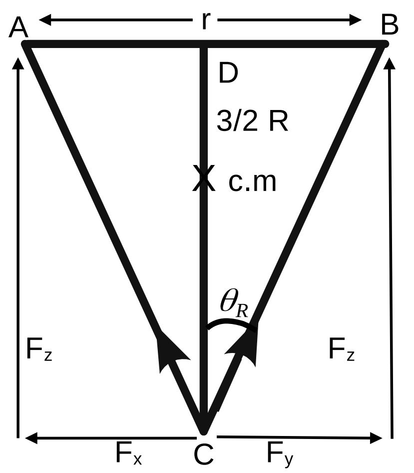

FIG. 3: Vector diagram for addition of forces F_{AC} and F_{BC} in terms of $F_{\parallel}$ and $F_{\perp}$.

the square root of the length. For the radially excited states one can have additional two quanta i.e. either n_r=1 or N_R=1, i.e. 2 $\hbar\omega_r$ or 2 $\hbar\omega_R$. This gives two possible 0^+ excited states the lower one will have n_r=0 and N_R=1 i.e. as $2\hbar\omega_R$ is $< 2\hbar\omega_r$. This can be associated with the Hoyle state. The second possibility is the higher energy 0^+ state with n_r=1 and N_R=0 as $2\hbar\omega_r > 2\hbar\omega_R$. This situation can be associated with the 9.93 MeV 0^+ excited state of ^{12}C.

The peturbing resonance phenomenon, i.e. expressing H in-terms of $\vec{R}_{AB}$ and $\vec{R}_{BC}$, can thus explain the occurance of two 0^+ excited states in ^{12}C. More detailed calculations can provide better estimates of the energies of Hoyle and second 9.93 MeV 0^+ state.

* Electronic address: arunjain45@gmail.com

[1] D, F. Jackson, Book, Nuclear Reactions. page 200 (1970)
[2] N. S. Chant and P. G. Roos, Phys. Rev. C **15** (1977) 57.
[3] A. K. Jain , Phys. Rev. C **45** (1994) 2387.
[4] B. N. Joshi *el al.*, DAE Symp NP. **68** (2024).