Study of cross sections and analyzing powers for nucleon-nucleon system with Pöschl–Teller potential plus spin dependence

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Introduction

Although electromagnetic spin-orbit effects are typically very small, of the order of one part in 10^5 in the spacing of atomic levels, it is not strong enough to support adequate changes in nuclear level spacing, required to generate the observed magic numbers. Therefore, we adopt the similar form as for the atomic spin-orbit concept but to nuclear potential. One gluon exchange (OGE) can be regarded as a viable source for velocity-dependent potential like spin-orbit coupling, which is exhibited by the strong polarization found in the scattering of nucleons by nuclei as well as by the substantial splitting between levels of a doublet explained in terms of a shell model. The precise dynamical origin of the strong nuclear spin-orbit force is not fully resolved, but the atomic physics analogy of the force hints to being a relativistic effect. Empirical data implies the dominance of static-type (velocity-independent) nuclear potentials at somewhat low energies for the incident particles.

The expectation value of the spin-orbit interaction is proportional to $2\langle L \bullet S\rangle = J(J+1) - L(L+1) - S(S+1)$ where the notations L, S and J stand for the orbital angular momentum, spin angular momentum and the total angular momentum respectively. Although traditionally the spin-orbit potential is a surface term and is proportional to $1/s\,(dV/ds)$, as its effect is piqued at the edge of the nucleus, and the spin density vanishes inside the nucleus. However, we adapt the Pöschl–Teller [1, 2] like spin-orbit term, added with the central Pöschl–Teller potential, hypothesizing that it will take care of the varying effects of the interior and surface of the nucleus with suitable parameterization of the potential. It is also well known that the form of the potential in spin-orbit term $V_{SO}(r)(L \bullet S)$ is not that vital, but it is the $(L \bullet S)$ factor which contributes significantly to reordering the levels. A significant spin-orbit coupling potential appears to be advantageous in explaining high-energy data.

Methodology:

The phase-function method (PFM) [1] is a powerful numerical technique that works well as an alternative to the traditional Schrödinger equation approach. Recent works by our group deal with this methodology in a somewhat lower energy region. The methodology is based on the possible reduction of second-order linear homogeneous equations to first-order nonlinear Riccati equations or phase equations, as given below.

$$\delta_\ell'(q,s) = -q^{-1}V(s) \times \left[\cos\delta_\ell(q,s)\,\hat{j}_\ell(qs) - \sin\delta_\ell(q,s)\hat{\eta}_\ell(qs)\right]^2 , \quad (1)$$

where $\hat{j}_\ell(qs)$ and $\hat{\eta}_\ell(qs)$ are the Riccati–Bessel functions. The resulting Phase equations for $\ell = 0,1 \& 2$ read as

$$\delta_0'(q,s) = -q^{-1}V(s)\left[\sin\left(\delta_0(q,s) + qs\right)\right]^2 , \quad (2)$$

$$\delta_1'(q,s) = -\frac{V(s)}{q^3 s^2} \times \left[\sin\left(\delta_1(q,s) + qs\right) - qs\cos\left(\delta_1(q,s) + qs\right)\right]^2 \quad (3)$$

and

$$\delta_2'(q,s) = -q^{-1}V(s)\left[\left(\frac{3}{q^2 s^2} - 1\right)\sin\left(\delta_2(q,s) + qs\right) - \frac{3}{qs}\cos\left(\delta_2(q,s) + qs\right)\right]^2 . \quad (4)$$

Where "q" stands for centre of mass momentum and is related to the centre of mass energy E as $q = \sqrt{2mE}\,/\hbar$. The term $\delta_\ell(q,s)$ is called the phase function, which satisfies the phase equation given by Eq. (1). With the initial condition $\delta_\ell(q,0) = 0$, phase equations given by Eqs. (2-4) are solved numerically for the potential under consideration to adjust the phase shifts of the scattering in different states in line with the established data.

The effective Pöschl–Teller potential [1, 2] in all partial waves is written as

$$V_{PT}(s)=\frac{V_1-V_2\ \cosh(\alpha s)}{\sinh^2(\alpha s)}+\frac{\ell(\ell+1)}{s^2} \quad (5)$$

and the effective spin orbit coupling is taken as

$$V_{SO}(s)=\left(\frac{V_1^{'}-V_2^{'}\ \cosh(\alpha s)}{\sinh^2(\alpha s)}\right)(L\bullet S). \quad (6)$$

The effective equivalent nuclear potential under investigation for uncharged hadrons is thus

$$V(s)=V_{PT}(s)+V_{SO}(s). \quad (7)$$

Result and Discussion:

To determine the standard phase parameters [3] of various states in the (n-p) system, we will parameterize the nuclear Pöschl-Teller potential, as given in Eq. (7), and solve the differential equations (2)-(4) numerically. For the numerical computations involving the (n-p) system, the parameters listed in Table 1 accurately reproduce the correct phase parameters up to a laboratory energy of 50 MeV. Our results align with those of Arndt et al. [3]. Using our phase parameters, we compute the scattering cross sections and analyzing powers for the system and compare these calculations with the existing data [4, 5] in the literature. Figure 1 displays the differential scattering cross sections for the (n-p) system at incident energies of 42.5 and 47.5 MeV, alongside the standard results from [4]. We present the analyzing powers for the (n-p) systems at $E_{Lab}=17.0\,MeV$ in Table 2 and compare them with the experimental data reported by J. Wilczynski et al. [5].

Table 1: List of parameters for the potential.

States (n-p)	α	V_1	V_2	$V_1^{'}$	$V_2^{'}$
$^3P_0$	0.82	5.02	2.85	0.32	1.02
$^3P_1$	1.32	8.01	3.86	3.62	0.33
$^3P_2$	0.82	6.05	2.88	1.65	0.99
$^3D_1$	1.03	8.65	2.02	5.01	0.85
$^3D_2$	1.05	5.05	11.32	2.36	6.33
$^3D_3$	1.05	5.06	8.33	1.96	4.75

As observed in Table 2, the A_y peak becomes higher and shifts backward with increasing energy. Additionally, both the peak height and the minimum of the cross section change gradually with energy. The discrepancy in the peak position may be attributed to the limitations of the PJ state nucleon-nucleon interaction within the model potential used for this calculation. Nevertheless, the overall agreement with our results remains satisfactory.

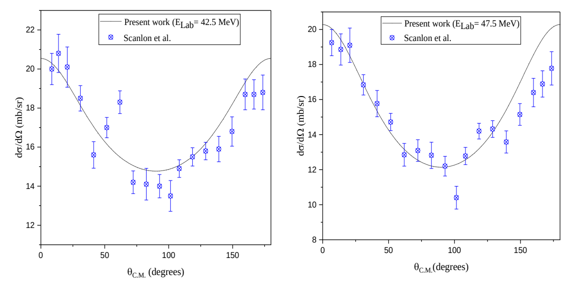

Fig. 1: Angular distributions for (n-p) scattering at energies of 42.5 and 47.5 MeV are presented, with standard data provided in Ref. [4].

Table 2: Neutron-proton (n-p) elastic analyzing power data at $E_{Lab}=17.0\,MeV$.

$\theta_{C.M.}$ (deg)	(Neutron-proton) $E_{Lab}=17.0\,MeV$	
	(Present work) A_y	Wilczynski et al. [5]
33.1	1.892440	1.90 ± 0.022
50.9	2.692307	2.43 ± 0.023
69.1	3.020172	2.95 ± 0.028
87.1	4.536850	4.52 ± 0.048
105.4	3.826081	2.99 ± 0.029
122.9	1.650562	1.34 ± 0.016

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