Global parameterization of nuclear level density parameters using Constant-Temperature Model

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Nuclear level density (NLD) plays a vital role in the statistical decay of excited nuclei, and is essential for calculating nuclear reaction cross sections. These cross sections are crucial for a range of applications, including astrophysical processes such as determining thermonuclear reaction rates for nucleosynthesis to the design of fusion and fission reactors. However, direct experimental determination of NLD is limited by the number of nuclear species and the range of excitation energies (E^*). Existing experimental data on NLD typically come from counting observed discrete states [1] or measuring NLDs near the neutron binding energy [2]. At low excitation energies, nuclear levels are sufficiently resolved with widths smaller than their spacing. When detailed spectroscopic studies are available, level densities can be inferred from tabulated observed levels. However, this method is limited by the accuracy and completeness of the data. Alternatively, neutron resonances can be used to infer the compound nuclear level density at neutron binding energy. But, this approach also comes with the limitations where observed levels exhibit a single parity and limited range of spin and are restricted to nuclei formed by capturing a single neutron onto the stable species.

Level densities are often required for many unstable nuclei, where direct experimental measurements are not feasible. In such cases, the theoretical models based on phenomenlogy or microscopic calculation are crucial. Microscopic models have made significant strides by properly incorporating shell effects, pairing correlations, and collective effects [3]. But, the complexity and computational time of these models restrict their practical applicability.

When the excitation energy of a nucleus is not significantly higher than the neutron binding energy, the basic phenomenological models with just two parameters for NLD, including the Constant-Temperature (CT) [4] and Back-shifted Fermi Gas (BSFG) [5] models, are often utilized to estimate the level density. These models are the extensions and modifications of the Fermi gas model and does not consider many details of the nuclear interactions. The model parameters are adjusted using the experimental level density data. These adjusted parameters are then utilized to develop a global prescription of NLD parameters that can be applied across different mass regions. In this study, we followed a similar approach using the Constant-Temperature model and in addition, incorporated the Bayesian optimization technique to derive a global prescription of NLD parameters based on measured NLD data. This process utilized the level density data from Oslo and particle evaporation measurements for a range of nuclei with mass number, $A \sim 40$ - 250.

The *Bayesian optimization* is a statistical technique that employs *Bayes'* theorem to optimize the model parameters based on experimental datasets. The posterior probability distribution of the model parameter is calculated using *Bayes'* theorem, which is expressed as:

$$P(M|D;\theta) = \frac{P(M;\theta)\ \mathcal{L}(D|M;\theta)}{\mathcal{Z}}, \qquad (1)$$

Here, $P(M;\theta)$ represents the *prior* distribution, which reflecting initial hypotheses, M about the model parameters, θ before considering the experimental dataset, D. The

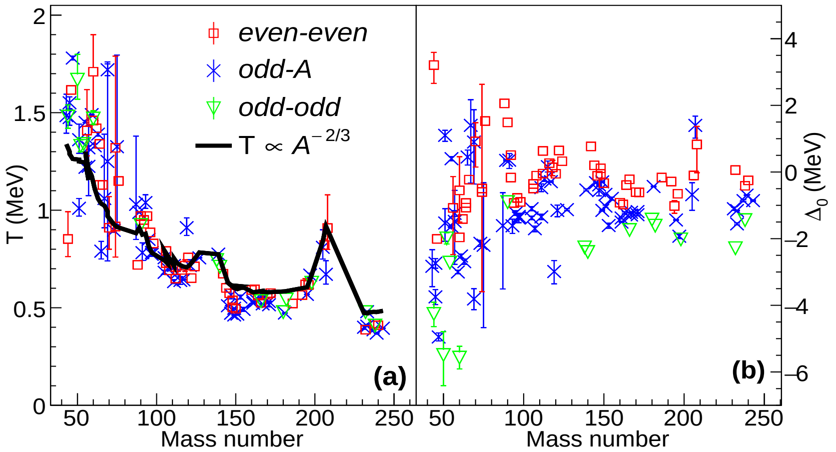

FIG. 1: Symbol represents empirical NLD parameters derived using Bayesian optimization techniques. The solid line represents theoretical models.

term $\mathcal{L}(D|M;\theta)$ is the *likelihood* function, which actually measure the compatibility of the observed data D with the given hypothesis M. In Eq.(1), the denominator $\mathcal{Z}[=\sum_i \mathcal{L}(D|M_i;\theta)P(M_i;\theta)]$ represents the *model evidence*, which is sum over all considered models. For this hypotheses M, we use the constant-temperature model [4] with nuclear level density: $\rho(E^*) = 1/T\ exp[(E^* - \Delta_0)/T]$. In the CT model, the nuclear temperature T and the energy-shift Δ_0 are treated as free parameters.

Figure 1 illustrates how the empirically determined NLD parameters (T, Δ_0) vary with the nuclear mass number (A). Parameter T does not exhibit any significant dependence on the nucleus type (even-even, odd-A, or odd-odd) but instead shows pronounced shell effects, with notable variations occurring at the shell closures. In contrast, the energy shift Δ_0 exhibit complex behavior with a clear dependency on the type of nucleus. Typically, these energies are less positive for even-even nuclei, smaller and negative for odd-A nuclei and more negative for odd-odd nuclei with distinct oscillatory patterns.

Inspection of Figure 1 reveals that parameter T exhibits an inverse dependence on the nuclear mass number. Several studies [6, 7] suggest that T typically shows an average mass dependence of $A^{-2/3}$. But, our detailed investigation reveals that nuclear temperature T actually follows a mass dependence of $A^{-8/9}$ rather than $A^{-2/3}$. This anomaly will be presented and discussed during the symposium.

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