Study of elastic neutron-^{12}C scattering using Phase Function Method

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Introduction

It is a well established fact that for light nuclei at low energies, the sharply defined resonance peaks reveal important information about the interaction between a nucleon and a nucleus [1]. The study of interaction between a neutron and ^{12}C isotope is very important since ^{12}C is an abundant isotope of carbon in nature ($\approx$ 98.9 %) and studying the interaction helps researchers to understand the nature of nucleon-nucleus interaction, the internal level structure of the ^{13}C isotope, binding energy and the nuclear reactions occuring in stars and the underlying nuclear forces occuring. It is also the final product of triple-alpha process occuring in the cores of stars, in which the three ^{4}He nuclei are fused to form a ^{12}C nucleus by releasing considerable amount of energy. One another interesting fact about ^{12}C is that it is a simple isotope of carbon with equal number of protons and neutrons, and is also quite stable with relatively very high binding energy per nucleon equal to 7.68 MeV. The ^{12}C plays a cruicial role in carbon-nitroge-oxygen (CNO) cycle also which is essential for understanding the life cycle of a star and and the energy production in red giant stars [2].
Interaction of neutron with ^{12}C nucleus (n-^{12}C) has been studied by many researchers experimentally and theoretically [1–3]. In this work, we are studying the interaction by determining scattering phase shifts and interaction potentials for s, p, d and f states. Experimental measurements for n-^{12}C in energy ranges from 1.45 to 7 MeV reveals resonances at neutron bombarding energies of 3.5, 4.23, 4.93 and 6.29 MeV. A Phase Function Method approach have been carried out to obtain the interaction potentials for n$-^{12}C$ elastic scattering for different channels in energy range 1.45-7 MeV [2, 3].

Methodology

The interaction between neutron and ^{12}C is modeled by considering Malfliet-Tjon (MT) [4] potential as the interaction potential. The MT potential is a three parameter potential consisting of both attractive and repulsive parts given as:

$$V(r) = -V_A\Big(\frac{e^{-\mu_A r}}{r}\Big) + V_R\Big(\frac{e^{-\mu_R r}}{r}\Big) \qquad (1)$$

with, $\mu_R = 2 \times \mu_A$. The interaction potential is incorporated in the phase equation [4], as:

$$\frac{d\delta(k,r)}{dr} = -\frac{V(r)}{k}\frac{2\mu}{\hbar^2}\Bigg[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr)\Bigg]^2 \qquad (2)$$

Here left side of equation presents differentiation of phase function with respect to distance. Also wavenumber $k_{cm} = \sqrt{\frac{E_{cm}}{\hbar^2/2\mu}}$; with $\mu_{n^{12}C}$ is the reduced mass of n-^{12}C system given by $(m_n * m_{^{12}C})/(m_n + m_{^{12}C}) = 866.81$ MeV/c^2 and $\hbar^2/2\mu$ for n-^{12}C system is equal to 22.46 MeV-fm^2. For higher partial waves (p, d & f) the Riccati-Bessel ($\hat{j}_\ell$) and Riccati-Neumann

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($\hat{\eta}_\ell$) functions can be obtained following recurrence formulas [5]

$$\hat{j}_{\ell+1}(kr) = \frac{2\ell+1}{kr}\hat{j}_\ell(kr) - \hat{j}_{\ell-1}(kr) \quad (3)$$

$$\hat{\eta}_{\ell+1}(kr) = \frac{2\ell+1}{kr}\hat{\eta}_\ell(kr) - \hat{\eta}_{\ell-1}(kr) \quad (4)$$

Results and Conclusion

In order to determine the interaction potentials for s, p, d and f states, the Riccati type first order non linear differential equation (NDE) Eq. 2 is solved numerically from origin ($\delta(0) = 0$) to asymptotic region. Fifth order Runge-Kutta (RK-5) method has been utised to solve NDE corresponding to different lab energies. The inverse potential parameters for MT potential have been obtained by minimising mean square error (MSE) corresponding to experimental values [2, 3] of scattering phase shifts δ_i^e given as:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(\delta_i^e - \delta_i^o)^2 \quad (5)$$

In Table I, the potential parameters V_r (MeV-fm), V_a (MeV-fm) and μ_A (fm^{-1}) have been given for Malfliet-Tjon potential. The parameters hace been optimised Variational Monte Carlo (VMC) technique [5] The scattering phase shifts corresonding to $s_{1/2}$, $p_{1/2}$, $d_{3/2}$ and $f_{7/2}$ states of $n - ^{12}C$ scattering are plotted in Fig. 1 and their respective potentials are given in Fig. 2.

TABLE I: Model parameters of MT potential for $s_{1/2}$, $p_{1/2}$, $d_{3/2}$ and $f_{7/2}$ states of $n - ^{12}C$ scattering.

States	$V_r(MeV)$	$V_a(MeV)$	$\mu_A(fm^{-1})$
$s_{1/2}$	15074.04	2051.96	0.56
$p_{1/2}$	408.05	614.31	5.98
$d_{3/2}$	7970.82	1981.83	1.28
$f_{7/2}$	14008.55	4905.97	2.13

It is to be noted in Fig. 1 that $s_{1/2}$ state phase shift δ_0 has been shifted by π i.e.,

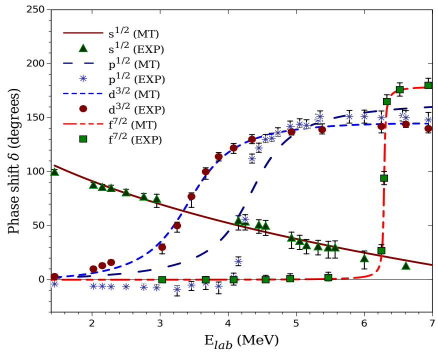

FIG. 1: SPS for $s_{1/2}$, $p_{1/2}$, $d_{3/2}$ and $f_{7/2}$ states of $n - ^{12}C$ scattering as a function of laboratory energy.

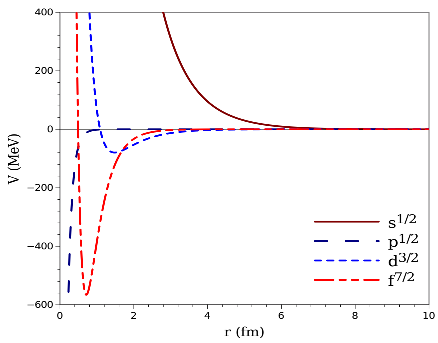

FIG. 2: $s_{1/2}$, $p_{1/2}$, $d_{3/2}$ and $f_{7/2}$ potentials of $n - ^{12}C$ scattering as a function of distance r.

$\delta_0 + \pi$ in order to show all phase shifts on the same π scale. Since the experimental data is very erroneous, we will follow a combinatorial data analysis and global optimization algorithm procedure to obtain a class of potentials that best suits $n - ^{12}C$ scattering, which will serve as our future prospect.

References

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