Decoding the Relationship Between Neutron Star Properties and Nuclear Matter Parameters Using Symbolic Regression

N. K. Patra[1],* Manish Kumar Sharma[1], and Harsh Chandrakar[1]
[1] *Department of Physics, BITS-Pilani, K. K. Birla Goa Campus, Goa 403726, India*

Introduction

The densest objects in the observable universe are neutron stars (NSs). Over the years, research into the equation of state (EOS) of NS matter has taken precedence in the fields of nuclear physics, high energy physics, and cosmology. The observed maximum neutron star mass, radius (R), and dimensionless tidal deformability (Λ) for canonical NSs ($1.4M_\odot$) are used to study the behaviour of the EOS at supra-saturation densities.

The EOS is probed by the NS properties throughout a wide range of masses and densities. At masses between (1.2 and 2.0) $M_\odot$, it is discovered that the values of R and Λ have a strong correlation with the density-dependent symmetry energy as well as with the pressure of β-equilibrium matter at densities almost twice the saturation density ($2\rho_0$). It is currently unclear how important nuclear matter parameters, like tidal deformability and radius at canonical mass 1.4 $M_\odot$, affect the properties of neutron stars. We use a symbolic regression method to look into the relationship between various nuclear matter properties (NMPs) and the tidal deformability and radius of neutron stars over a broad mass range. For this analysis, we use the relativistic mean-field (RMF) model outlined in Ref. [1] to analyze the nuclear matter equation of state.

EOS and its Parameters

The Lagrangian density for the RMF model is given in Ref. [1]. It contains non-linear meson terms, including both self-interactions and mixed mesonic terms. The energy per nucleon $\epsilon(\rho, \delta)$ at given density and asymmetry(δ) can be decomposed as,

$$\epsilon(\rho, \delta) \;=\; \epsilon(\rho, 0) + S(\rho)\delta^2 + ..., \qquad (1)$$

Here, the isospin asymmetry $\delta = \frac{\rho_n - \rho_p}{\rho}$ measures the neutron-proton difference in nuclear matter. ϵ represents the energy per nucleon at a given density ρ. At saturation density (ρ_0), the equation of state (EOS) can be expressed through different bulk nuclear matter properties of order n. Specifically: (i) For symmetric nuclear matter, the energy per nucleon is $\mathbf{e} = \epsilon(\rho_0, 0)$ ($n = 0$), the incompressibility is K_0 ($n = 2$), skewness is Q_0 ($n = 3$), and kurtosis is Z_0 ($n = 4$). (ii) For symmetry energy, the symmetry coefficient is $J_0 = S(\rho_0)$ ($n = 0$), the slope is L_0 ($n = 1$), curvature is $K_{sym,0}$ ($n = 2$), skewness is $Q_{sym,0}$ ($n = 3$), and kurtosis is $Z_{sym,0}$ ($n = 4$).

$$\chi_0^{(n)} \;=\; 3^n \rho_0^n \left(\frac{\partial^n \epsilon(\rho, 0)}{\partial \rho^n}\right)_{\rho_0}, n = 2, 3, 4; \quad (2)$$

$$\chi_{\rm sym,0}^{(n)} \;=\; 3^n \rho_0^n \left(\frac{\partial^n S(\rho)}{\partial \rho^n}\right)_{\rho_0}, n = 1, 2, 3, 4; \quad (3)$$

Results

For this work, we employ datasets generated through Bayesian inference in conjunction with precise theoretical calculations for pure neutron matter pressure at low densities and empirically known few nuclear saturation properties. Our dataset contains all nuclear matter parameters defined at saturation density, along with neutron star properties such as radius and tidal deformability at mass 1.4 $M_\odot$.

We added an issue that there is a lack of robustness in the correlations of NS properties with nuclear matter parameters. To address this issue, We use a symbolic regression method, a type of

*Electronic address: nareshkumarpatra3@gmail.com

TABLE I: The optimal equations are presented. Where Y exclusively represents the radius (R) and tidal deformability (Λ) at neutron star mass 1.4 $M_{\odot}$. Pearson's correlation coefficients and the root mean square error (RMSE) in relation to mean values are also provided.

Y	**Equations**	**Correlation**	**RMSE (in %)**
$R_{1.4}$	$0.0208 \times K_0 + 0.1237 \times L_0 + 0.019 \times K_{\rm sym,0} + 0.0026 \times Q_{\rm sym,0}$	0.80	8.2
$\Lambda_{1.4}$	$0.99 \times K_0 + 4.46 \times L_0 + 0.87 \times K_{\rm sym,0} + 0.067 \times Q_{\rm sym,0}$	0.83	7.6

machine learning algorithm among others, to analyze the relations among data. We first extract the characteristic vectors $\mathbf{X}[\mathbf{n}]$ and target vector $\mathbf{Y}[\mathbf{n'}]$ from the dataset. A random subset of features **n1** and a single target are then selected and fed into the GPlearn algorithm. The best equation is identified based on the correlation coefficient and root mean square error with the original data.

In Table I, we presented the relationships between NS properties such as radius, tidal deformability, and specific nuclear matter parameters for neutron stars. It is interesting to notice that some NMPs, including $\mathbf{e_0}$, $J_{sym,0}$, and $Z_{sym,0}$, have no effect on both radius and tidal deformability. The table also illustrates the collaborative role of both iso-scalar and iso-vector components of the EOS in determining the radius and tidal deformability. However, the influence of nuclear matter parameters, corroborating findings from prior research by Patra et al. [2].

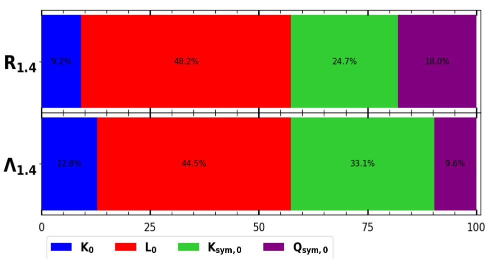

FIG. 1: The values of the percentage contributions of nuclear matter parameters to the radius and tidal deformability of neutron star at mass $1.4 M_{\odot}$ (refer to Table-I).

Figure 1 shows the results of a PCA used to evaluate how different nuclear matter parameters contribute to neutron star properties at a mass of 1.4 $M_{\odot}$ (see Table I). The two horizontal bars represent the radius ($R_{1.4}$) and tidal deformability ($\Lambda_{1.4}$). These bars are divided to show the percentage contributions from five nuclear matter parameters: incompressibility (K_0), skewness (Q_0), the slope of symmetry energy (L_0), the curvature of symmetry energy ($K_{sym,0}$), and the higher-order term ($Q_{sym,0}$). These parameters form the equation of state for nuclear matter, which determines neutron star properties. The graph reveals that L_0 has the largest impact on the radius ($R_{1.4}$), while $\Lambda_{1.4}$ is most sensitive to L_0 and $K_{sym,0}$.

Conclusion

We have shown that neutron star properties vary based on several nuclear matter parameters, using symbolic regression, a type of machine learning algorithm. Our results indicate that the symmetry energy parameters, specifically the slope (L_0) and curvature ($K_{\rm sym,0}$), play a dominant role in determining the neutron star's radius and tidal deformability, respectively. As neutron star mass increases, the iso-scalar nuclear matter parameters, such as the incompressibility coefficient (K_0) and skewness (Q_0) of symmetric nuclear matter, become more influential. Notably, the contribution of K_0 becomes significant for neutron stars with masses above 1.8 $M_{\odot}$.

References

[1] T. Malik *et al.*, [arXiv:2301.08169] (2023)
[2] N. K. Patra *et al.*, [arXiv:2308.13896](2023).