Influence of Dark Matter on Neutron Star Curvature within the Quarkyonic Model: A Relativistic Mean Field Approach

Jeet Amrit Pattnaik[1],* D. Dey[2,3], R. N. Panda[1], M. Bhuyan[2], and S. K. Patra[1]
[1] *Department of Physics, Siksha 'O' Anusandhan,*
Deemed to be University, Bhubaneswar-751030, India
[2] *Institute of Physics, Sachivalya Marg, Bhubaneswar-751005, Odisha, India and*
[3] *Homi Bhabha National Institute, Training School Complex,*
Anushakti Nagar, Mumbai 400094, India

Introduction

The study of neutron stars (NS) presents one of the most intriguing challenges in astrophysics, primarily due to the extreme conditions that cannot be recreated in laboratories. NSs, the remnants of massive stars that have undergone supernova explosions, possess extremely high densities, strong gravitational fields, and compact structures, making them ideal astrophysical objects for testing the properties of matter under such conditions. Astrophysical observations and theoretical models suggest that neutron stars may contain exotic forms of matter such as hyperons, pion or kaon condensates, quarks, and possibly dark matter (DM) [1]. The presence of DM, which constitutes a large portion of the universe's mass, is particularly interesting because it could significantly influence the structure and observable properties of neutron stars. Various models have been proposed to explain the interaction between neutron stars and dark matter, including the quarkyonic matter model [2], which suggests that nucleons transition into quarks at intermediate baryon densities. This model aligns with earlier speculations about the presence of quark matter in NSs. The interaction between dark matter and standard matter, mediated by Higgs bosons, can be effectively modeled using the relativistic mean-field (RMF) theory, particularly with the star's curvature. Curvature, as described by Einstein's general theory of relativity, quantifies the warping of spacetime caused by massive objects like neutron stars. The presence of DM is expected to soften the EoS, leading to a reduction in the star's mass and radius. Moreover, DM could significantly affect the surface curvature of neutron stars, especially in massive stars, making them key objects of study for testing the limits of general relativity.

Formalism

The E-RMF model-based IOPB-I parameter set is used for the study of quarkyonic star with DM inside it. For this, we constructed the Lagrangian density consisting of the baryonic, quark, and dark matter, respectively [3]. The Lagrangian density is followed by:

$$\mathcal{L} = \mathcal{L}_{\mathcal{RMF}} + \mathcal{L}_{\mathcal{QM}} + \mathcal{L}_{\mathcal{DM}}, \tag{1}$$

where $\mathcal{L}_{\mathcal{RMF}}$,

$$\mathcal{L}_{\mathcal{QM}} = \textstyle\sum_{j=u,d} \bar{\psi}_j (i\gamma_\nu \partial^\nu - m_j)\psi_j, \tag{2}$$

$$\begin{aligned}\mathcal{L}_{\mathcal{DM}} &= \bar{\chi}\left[i\gamma^\mu\partial_\mu - M_\chi + yh\right]\chi + \tfrac{1}{2}\partial_\mu h \partial^\mu h \\ &\quad -\tfrac{1}{2}M_h^2 h^2 + f\tfrac{M_{nucl.}}{v}\bar{\varphi}h\varphi\end{aligned} \tag{3}$$

Mathematical formulation for different curvatures

For both inside and outside the star, we consider different curvature quantities from [4]. The curvatures are Ricci scalar, Ricci tensor, Riemann tensor, and Weyl tensor, are defined as follows;
The Ricci scalar

$$\mathcal{R}(r) = 8\pi\Big[\mathcal{E}_{tot.}(r) - 3P_{tot.}(r)\Big], \tag{4}$$

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[*] Electronic address: jeetamritboudh@gmail.com

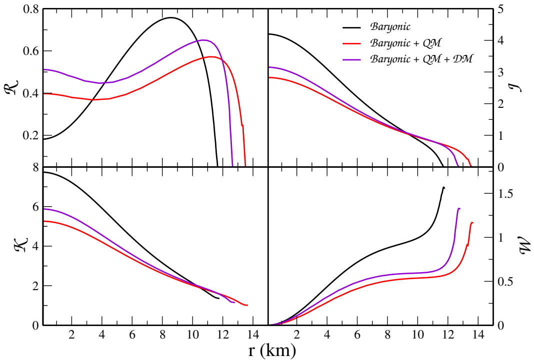

FIG. 1: Various curvatures like $\mathcal{R}$, $\mathcal{J}$, $\mathcal{K}$, and $\mathcal{W}$ of the maximum mass NS (2.15 $M_\odot$, 2.50 $M_\odot$, and 2.34 $M_\odot$ respectively) for baryonic, quarkyonic ($n_t = 0.3 fm^{-3}$, $\Lambda_{cs} = 800$ MeV) and DM admixed quarkyonic cases($k_f^{DM} = 0.03$ GeV) for the IOPB-I parameter set.

the full contraction of the Ricci tensor

$$\mathcal{J}(r) = \left[(8\pi)^2\left[\mathcal{E}^2_{tot.}(r) + 3P^2_{tot.}(r)\right]\right]^{1/2}, \quad (5)$$

the Kretschmann scalar (full contraction of the Riemann tensor)

$$\mathcal{K}(r) \equiv \sqrt{\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}}, \quad (6)$$

and the full contraction of the Weyl tensor

$$\mathcal{W}(r) = \left[\frac{4}{3}\left(\frac{6m(r)}{r^3} - 8\pi\mathcal{E}_{tot.}(r)\right)^2\right]^{1/2}. \quad (7)$$

Where $\mathcal{E}_{tot.}$, $P_{tot.}$, $m(r)$ and r are the energy density, pressure, mass and radius of the NS respectively.

Results

In Fig. 1, we have shown the variation of different curvatures with the radial component of maximum mass NS for baryonic, quarkyonic, and DM admixed quarkyonic stars. At the center of the star, all the curvatures have the maximum values contradictory to the Weyl tensor. The addition of quarkyonic matter (red curve) results in notable changes in the curvature trends, especially in the inner regions. Despite the stiffer equation of state compared to the baryonic case (black curve), the curvatures exhibit a softer trend, meaning they decrease more rapidly with radius. This is because stiffer equations of state lead to larger radii for neutron stars, which in turn reduces the curvature. When dark matter is introduced to quarkyonic stars (violet curve), the equation of state softens, but the curvatures show a stiffer trend compared to quarkyonic stars alone [3]. This reflects a balance between the influence of DM and the still stiffer nature of the EOS relative to purely baryonic stars. The overall behaviour shows that while the EOS stiffens with the addition of quark matter and softens with dark matter, the curvature trends are the opposite, with softer EOS leading to stiffer curvatures.

Conclusion

This study explores the impacts of DM on the curvature of quarkyonic stars using an E-RMF-based IOPB-I parameter set. Further, the study of curvature variation with neutron star radius for baryonic, quarkyonic, and dark matter admixed quarkyonic stars reveals a clear inverse relationship between the stiffness of the equation of state (EOS) and the curvature trends. This inverse relationship between EOS stiffness and curvature trends shows the complex interaction between matter composition and the structure of neutron stars, emphasizing how quark matter and dark matter affect the star's compactness and curvature.

Acknowledgments

This work has been supported by SERB Ramanujan Fellowship File No. RJF/2022/000140.

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