Dark Matter Effects on $f-$mode Oscillations in Quarkyonic Stars

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1. INTRODUCTION

Neutron stars (NS) are incredibly dense and compact celestial objects, with radii around 10-15 km and masses about 2 $M_\odot$. Their extreme conditions arise from supernova explosions, and their internal composition is a subject of intense study. The relativistic mean-field (RMF) formalism is a robust theoretical framework for describing both finite nuclei and extremely dense nuclear matter. Despite our inability to recreate these extreme conditions in laboratories, astrophysical observations and theoretical models offer valuable insights. The interior of neutron stars is likely composed of various subatomic particles and exotic states of matter, including free quarks, hyperons, pion condensation, and possibly dark matter. The quarkyonic matter model describes a crossover transition between nucleons and quarks in neutron stars [1]. We consider fermionic dark matter interacting with nucleons via Higgs boson exchange [2]. Gravitational waves (GW) reveal neutron star properties through quasinormal modes (QNM), with the fundamental f mode [3] providing insights into the interior composition and damping forces. These modes, including $p-$, $g-$, and $r-$ modes, are excited during supernovae, starquakes, or neutron star mergers. The $f-$mode is notable for its detection by current GW instruments and its correlation with tidal deformability, while the $g-$ modes are too weak for current detection. This study calculates the $f-$modes of dark matter-admixed quarkyonic stars [4] using the Cowling approximation where metric perturbations are neglected.

2. FORMALISM

The E-RMF model-based IOPB-I set is used for the quarkyonic neutron star with DM inside it. For this, we constructed the Lagrangian density consisting of the baryonic (BM), quarkyonic (QM), and dark matter (DM), respectively

$$\mathcal{L} = \mathcal{L}_{\mathcal{RMF}} + \mathcal{L}_{\mathcal{QM}} + \mathcal{L}_{\mathcal{DM}}, \quad (1)$$

where $\mathcal{L}_{\mathcal{RMF}}$ is the baryonic lagrangian density and

$$\mathcal{L}_{\mathcal{QM}} = \textstyle\sum_{j=u,d} \bar{\psi}_j (i\gamma_\nu \partial^\nu - m_j)\psi_j, \quad (2)$$

$$\begin{aligned}\mathcal{L}_{\mathcal{DM}} &= \bar{\chi}\left[i\gamma^\mu \partial_\mu - M_\chi + yh\right]\chi + \tfrac{1}{2}\partial_\mu h \partial^\mu h \\ &\quad -\tfrac{1}{2}M_h^2 h^2 + f\tfrac{M_{nucl.}}{v}\bar{\varphi} h \varphi \quad (3)\end{aligned}$$

the equations describing the nonradial perturbations of the stars Cowling approximation is given by

$$\begin{aligned}\frac{dW}{d\ln r} &= -(l+1)\left[W - l e^{\nu+\lambda/2} U\right] \\ &\quad - \frac{e^{\lambda/2}(\omega r)^2}{c_{ad}^2}\left[U - \frac{d\Phi}{d\ln r}\frac{e^{-\lambda/2}}{(\omega r)^2} W\right], \quad (4) \\ \frac{dU}{d\ln r} &= e^{\lambda/2-\nu}\left[W - l e^{\nu-\lambda/2} U\right] \\ &\quad + \Delta(c^{-2})\frac{d\Phi}{d\ln r}\left[U - \frac{d\Phi}{d\ln r}\frac{e^{-\lambda/2}}{(\omega r)^2} W\right], \quad (5)\end{aligned}$$

$$\begin{aligned}\left.\frac{W}{U}\right|_{r=0} &= l e^{\nu_c} = l e^{2\Phi_c} \quad (6) \\ \left.\frac{W}{U}\right|_{p=0} &= \frac{e^{\lambda/2}(\omega r)^2}{\frac{d\Phi}{d\ln r}} = \frac{(\omega r)^2\sqrt{1-2b}}{b+4\pi r^2 p}\Big|_{p=0} \\ &= (\omega R)^2 \frac{c^2 R}{GM}\sqrt{1-\frac{2GM}{c^2 R}}, \quad (7)\end{aligned}$$

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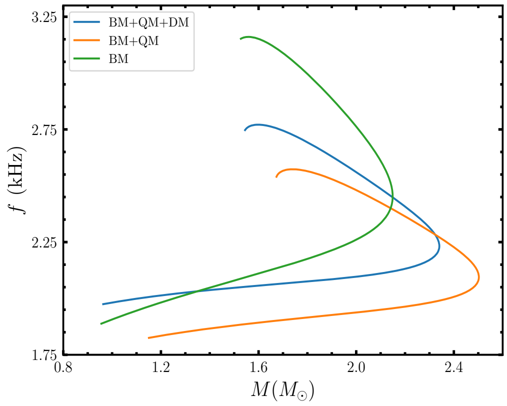

FIG. 1: (Color online) $f-M$ relations for (a) BM (b) QM ($n_t = 0.3 fm^{-3}$, $\Lambda_{cs} = 800$ MeV) (c) DM admixed QM ($k_f^{DM} = 0.03$ GeV) for IOPB parameter set.

The equations are solved for l =2 starting from the center and extending outward to the surface, aiming to satisfy the boundary conditions at the surface.

3. RESULTS

The Fig.1 illustrates the frequency-mass (f-M) relationship for neutron stars under different compositions: BM, BM+QM, and DM admixed quarkyonic matter (BM+QM+DM), based on the IOPB parameter set. The green curve represents BM, showing the variation of frequency with mass when only baryonic matter is considered. The blue curve corresponds to BM+QM, where quarkyonic matter is included at a transition density $n_t = 0.3\ fm^{-3}$ and confinement scale Λ_{CS} =800MeV. The orange curve represents BM+QM+DM, where dark matter with momentum k_f^{DM}=0.03GeV is added to the quarkyonic matter component. Table I accompanying the plot quantifies the maximum mass and associated frequencies for each scenario, showing that the inclusion of quarkyonic matter lowers the maximum frequency while mixing of DM makes the star less compact and hence raises the frequency.

4. CONCLUSIONS

We studied $f-$mode oscillations of neutron stars with dark matter using the quarkyonic model and IOPB-I parameter set. Our findings reveal how dark matter affects oscillation frequencies and neutron star structure, offering insights into exotic matter and aiding future gravitational wave detections for refining neutron star equations of state.

TABLE I: Table for maximum mass, corresponding radius and canonical star for the above-mentioned EOS at $n_t = 0.3 fm^{-3}$, $\Lambda_{cs} = 800$MeV with DM momentum $k_f^{DM} = 0.03$ GeV.

	BM	BM+QM	BM+QM+DM
$M(M_\odot)$	2.15	2.50	2.34
f (kHz)	2.45	2.08	2.22

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