Stellar Energy Generation and the Future of n-free Fusion

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Introduction

Since the early 20th century, astrophysics has been concerned with explaining how energy is generated in stellar interiors through thermonuclear reactions within a plasma at temperatures reaching hundreds of millions of degrees Kelvin. These processes, including hydrogen burning chains and the CNO cycle, begin with hydrogen nuclei. Over the past century, nuclear astrophysicists have aimed to model the elemental abundances within stars. The nature of nuclear reactions means that multiple processes can occur under the same conditions. For stellar nucleosynthesis, by-product reactions, particularly those involving neutron emission, are of academic interest, but they pose hazards in practical applications because neutrons can induce radioactivity in materials. To avoid such hazardous by-products, reactions that require higher energy for neutron emission are preferred. One notable aneutronic process is the fusion of boron-11 (^{11}B) with protons (p) to produce carbon-12 (^{12}C), which then breaks into three alpha particles [1]. This process, which has a neutron emission threshold of about 3 MeV proton energy (or a temperature above 30 billion Kelvin), is cleaner and safer as it avoids neutron production, making it an attractive option despite the practical and economic challenges.

Basic physics of Nucleosynthesis

It is important to note that most nucleosynthesis occurs through interactions between two positively charged particles, which are influenced by Coulomb repulsion. For neutral particles, one must consider the repulsive centrifugal barrier. According to kinetic theory, the kinetic energy (K.E.) of a particle is related to the temperature (T) of the medium by the equation K.E. = k_B*T, where k_B = 86 μeV/K is the Boltzmann constant. Typically, temperatures are expressed in gigakelvin (T_9 = 86 keV).
The nucleosynthesis is predominantly proceeded by two factors, the quantum tunnelling and the number distribution of the particles at a fixed temperature. Assuming the Maxwell-Boltzmann distribution of kinetic theory, the synthesis yields are given by the Gamow window with maximum at energy [2]
$E_0(MeV) = 0.122\ (\mu_A Z_1^2 Z_2^2 T_9^2)^{1/3}$; and spread of $\Delta E\ (MeV) = 0.2368(\mu_A Z_1^2 Z_2^2 T_9^5)^{1/6}$, $\mu_A = \frac{m_1 m_2}{m_1+m_2}$ is the reduced mass (in dimensionless atomic mass units) of two interacting nuclei of masses and charge states of (m_1, Z_1) and (m_2, Z_2), respectively. T_9 is the temperature in Giga Kelvin units.

Limitation: The formulas derived using a Maxwellian distribution assume that particles are ideal, non-interacting gas molecules and that one of the interacting partners is at rest. However, this is not the case in reality. In astrophysical conditions, both particles are typically in motion and move randomly with respect to each other, leading to some energy being used up in phase space conservation. For a process with a positive Q value, the maximum energy released to the external world is given by $E_{max} = Q\frac{m_2}{m_1+m_2}$ where particle 2 (the target) is at rest and particle 1 (the projectile) is moving. This formula accounts for the fact that the center of mass is not stationary. An exception occurs when one of the interacting partners is a photon, in which case the center of mass is effectively at the heavy baryon. To make much more efficient energy sources for terrestrial applications, some modification can be made to utilize the max of the energy from the nuclear reaction [3].

Modification: It is well-established that Boltzmann entropy and the Maxwell-Boltzmann distribution apply to systems in thermal equilibrium, while dynamical processes are inherently non-equilibrium. Additionally, space plasma is characterized by a multi-Maxwellian distribution, where a single Gaussian approximation is inadequate. To accurately model multi-Gaussian distributions, Kappa and Levy statistics are used. Although the theoretical treatments involve complex mathematical manipulations, they ultimately reduce to multiple Gaussian distributions. In the Kappa distribution, the definitions of physical and kinetic temperatures are: [4]

$$T_{phys} = \left({}^{\partial S}/_{\partial U}\right)^{-1} \left[1 - \frac{1}{\kappa} * \frac{S}{k_B}\right];\quad T_{kin} = \frac{2U}{n * k_B}$$

Where, S represents entropy, U denotes the kinetic energy of particles, k_B is the Boltzmann constant, and κ is the Kappa distribution index, with n indicating the number of degrees of freedom. As κ approaches infinity, the physical definition of temperature converges to the equilibrium condition, while the kinetic temperature remains consistent with the classical definition. Exploring how parameters affect the dynamics of plasmas with multiple ion species is both interesting and valuable. Plasma experiments for energy production will serve as testing grounds to evaluate the significance of these theoretical extensions, which have implications for stellar nucleosynthesis and our broader understanding of cosmology.

Regardless of the fusion reactor technology used (such as tokamaks or laser confinement), the goal is to bring the two interacting partners together to achieve a center of mass energy (E_{cm}) sufficient to trigger the desired reaction. The kinematics of this process can be summarized by: $E_{cm}^2 = (E_1 + E_2)^2 - \left(\overrightarrow{P_1} + \overrightarrow{P_2}\right)^2$, where Es are energies (mass +kinetic energy) and Ps are 3-vector momenta of the interactants. Clearly, the center of mass energy is maximum when the interactants are of equal and opposite momenta. In stellar atmospheres, random motions of ions of both interacting species would render the available energy of the system within limits of $(E_1 + E_2)^2 - (P_1 + P_2)^2 \le E_{cm}^2 \le (E_1 + E_2)^2 - (P_1 - P_2)^2$
For non-equal masses, momenta are not equal at a fixed temperature T. In laboratory settings, head-on collisions can be arranged so that the net momentum is zero, ensuring that the entire energy is available for release. To accomplish this, the ions need to be moving in opposite directions with equal momenta. In the ideal gas approximation, the kinetic energy (K.E.) is given by is $k_{B*}T$ and the momentum is $\sqrt{2mk_BT}$ in non-relativistic approximation, an excellent approximation for fusion phenomena since the interacting partners are of very low kinetic energies. This implies that for two interacting ions of different masses, the optimization of the energy-momentum requires that the ions be at different temperatures. More precisely, the required temperatures are inversely proportional to ion masses. If we operate them at the same temperature, part of the energy is wasted in the phase-space as the vector momentum of the center of mass. With the optimized temperatures ($m_1T_1 = m_2T_2$), the center of mass energy is given by, $E_{cm} = k_B(m_1T_1 + m_2T_2) = 2k_Bm_1T_1 = 2k_Bm_2T_2$

Conclusion: The concept of the Gamow window, used in astrophysical nucleosynthesis, is difficult to test directly. Given its importance for sustainable nuclear fusion energy, an experimental program focused on some of the reactions like (^{11}B+p), (^{3}He+d), both in forward (lighter projectile) and inverse (heavier projectile) conditions, is necessary to determine reaction yields. This would help evaluate the influence of non-Maxwellian and Kappa distributions, improving the design of plasma devices for energy generation and enhancing astrophysical model accuracy.

References

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