Non-Radial oscillations in Neutron stars with Dark matter Core

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Introduction

Neutron stars are possible remnants of very massive star, left behind after supernova explosions. These stars are incredibly compact, believed to have a radius of about $(10-15)$ km and can have masses up to 2 solar mass. They have intense magnetic fields and can spin rapidly, emitting beams of radiation, which are detected and hence coined the name "pulsars". Almost about 3500 of these exotic species have been observed till date and with advances in the Gravitational wave astronomy, the era of multi-messenger has arrived.

These compact stars are subject to disturbances, which can be seen in terms of the radial or non radial-modes. In radial oscillations, a star expands and contracts around its configuration, maintaining its spherical shape throughout. In contrast, the non-radial mode, such as fundamental (f-mode), pressure (p-mode), and gravity (g-mode) cause distortion to the star's shape. The f-modes are really interesting because they create gravitational radiation that can be picked up by current observatories. For example, in the event GW170817, the f-mode frequency for the heavier neutron star falls between 1.43 kHz and 2.90 kHz with 90% certainty, while for the lighter ones, it ranges from 1.48 kHz to 3.18 kHz. These oscillations (f-mode) in neutron stars have been largely investigated using the Cowling approximation, which neglects background spacetime perturbations. The results were refined to include full general relativistic (GR) effects. Several studies have explored f-modes in neutron stars with dark matter admixtures. For the dark matter model, we are motivated by the neutron decay anomaly model of the dark matter. This study focuses on the non-radial oscillations of neutron star (f-mode and p-mode) using relativistic Cowling approximation.

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Methodology

Our calculation of non-radial mode oscillations relies on the Cowling approximation, which ignores perturbations in the background metric.

In this approximation, the spacetime metric for a spherically symmetric star is given by:

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

To find the mode frequencies, we solve the following differential equations:

$$\frac{dW(r)}{dr} = \frac{d\epsilon}{dp}\left(\omega^2 r^2 e^{\Lambda(r)-2\Phi(r)}V(r) + \frac{d\Phi(r)}{dr}W(r)\right) - l(l+1)e^{\Lambda(r)}V(r)$$

$$\frac{dV(r)}{dr} = 2\frac{d\Phi(r)}{dr}V(r) - \frac{1}{r^2}e^{\Lambda(r)}W(r)$$

Here, $\frac{d\Phi(r)}{dr}$ is given by:

$$\frac{d\Phi(r)}{dr} = -\frac{1}{\epsilon(r)+p(r)}\frac{dp}{dr}$$

The solution near the center of the star behaves as:

$$W(r) = Ar^{l+1}, \quad V(r) = -\frac{A}{l}r^l$$

At the surface, the vanishing of the perturbed Lagrangian pressure imposes the boundary condition:

$$\omega^2 e^{\Lambda(R)-2\Phi(R)}V(R) + \frac{1}{R^2}\left.\frac{d\Phi(r)}{dr}\right|_{r=R} W(R) = 0$$

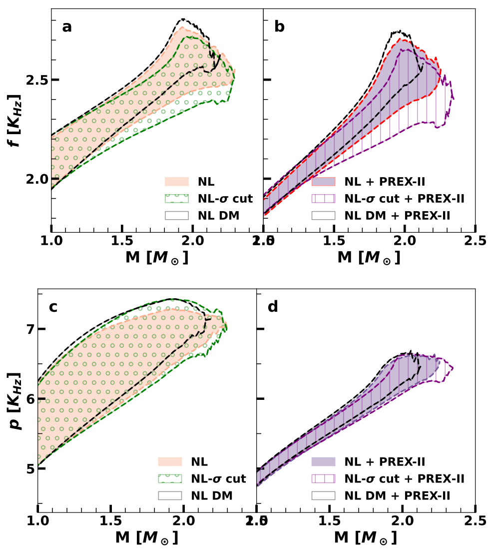

FIG. 1: The frequencies of non radial modes, f mode (upper plot) and p mode (lower plot) versus the neutron star mass.

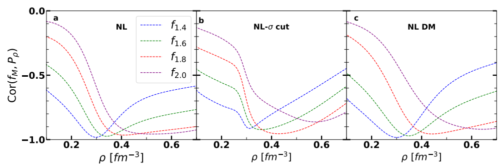

FIG. 2: This graph indicates the Pearson's correlation coefficients between the f-mode oscillation frequency f_M for neutron star masses and the neutron star matter pressure $P(\rho)$ for models without the PREX-II

These differential equations are eigenvalue problems, and the values of ω that satisfy this boundary condition are the star's eigenfrequencies.

Results & Conclusions

We have calculated the frequencies of the non-radial oscillation mode (f and p-modes) of neutron stars using Cowling approximation. This analysis is based on the posterior results from three cases: NL, NL-σ cut [2] and NL-DM.

Figure 1 shows the f-mode and p-mode frequencies of neutron stars as functions of their mass. As the neutron star mass increases from $(1.0 - 2.3)$ solar masses, the f-mode frequency are approximately from $(2.0 - 2.8)$ kHz and from $(5 - 7.5)$ kHz over the same mass range. For PREX-II, narrower distribution at lower neutron masses can be seen. The slope for f-mode varies for the three cases [1]. The slope for p-mode shows only slight variation across three cases. The f-mode is more sensitive to the symmetry energy and provides similar slope information as mass radius relationship.

Figure 2 shows the Pearson correlation coefficients between pressure and f-mode frequencies of neutron star of different masses, which are plotted against the baryon number density. The correlations are the strongest in the NL-model, but are fairly similar across all models. For NL-σcut model, adding the σ cut potential seems to weaken the correlation for lower mass stars. In the NL-DM model, the correlation is more affected in the heavier stars when the impact of dark matter is stronger.

We would further like to investigate the same with $\sigma-$ cut approach within effective chiral model in near future [3]. Some preliminary results will be shown.

[1] P. Thakur et. al., *arXiv:2408.03780 [nucl-th]* (August 2024).

[2] P. Thakur et. al., *Phys. Rev. C*, **109**(2): 025805, 2024.

[3] Harsh Chandrakar and T. K. Jha, **(under preparation)**, 2024.