Superfluidity: Cold Atoms to Neutron Stars - corrections to self-energy

S. Ramanan[1]* and M. Urban[2]
[1]*Department of Physics, IIT Madras, Chennai, India and*
[2]*Université Paris-Saclay, CNRS-IN2P3, IJCLab, 91405 Orsay, France*

Introduction

Neutron stars presents a rich system of asymmetric nuclear matter, infinite in extent, that is held is together by gravity, ranging from nuclei at terrestrial densities at the outermost layers, to neutron rich nuclei and neutron superfluid interspersed between a lattice of quasi-nuclei clusters to finally a core, whose composition is still under investigation. A complete understanding of the physics of the crust and the outer layers of the core of the neutron star remains far from satisfactory due to the uncertainties in the nuclear interaction that is strongly modified by the nuclear medium. As a first approximation, neutron rich matter in the inner layers of the crust, it is modelled as a uniform gas of neutrons, ignoring the effect of the lattice. Neutron gas shares

While a large scattering length naturally occurs in nuclear systems, in a laboratory, it is possible to tune the scattering length of an atomic gas of ultracold fermions to very large values via Feshbach resonances and in this respect, the neutron gas and cold atomic fermions are very similar. The simplicity of atomic interactions makes for an ideal test bed of various theoretical approaches that are relevant to neutron star physics. In fact, ultracold fermi gas can be considered as a quantum simulator for a neutron star.

In cold atomic systems, the effective range is several orders of magnitude smaller than the scattering length and as a result, and hence it is customary to model these systems via a contact interaction. However, a contact interaction requires infinite resummation in a many-body calculation. In a recent work [1] that studied the equation of state of a gas of ultracold fermions, the renormalization group approach developed in low-energy nuclear structure to obtain effective interaction, was adapted to the system of cold fermi gas. This effective interaction has an explicit dependence on the renormalization scale (RG) λ, which when lowered, results in interactions that are amenable to perturbation theory. However, properly renormalized physical observables should not depend on λ and any dependence on this scale indicates the dependence of missing medium and/or higher-body terms.

In this talk, we extend the study in [1] and examine the corrections to the self-energy (normal and anomalous contributions). The input interactions are the low-momentum effective interaction constructed to preserve the two-body phase shifts of the ultracold fermi gas, with an explicit dependence on the resolution scale λ. In Sect. 1, the interaction and our approach to incorporating perturbative corrections are outlined, while Sect. 2 presents our preliminary results and future directions.

1. Formalism

The Hamiltonian

$$H = H_0 - \mu N + H_{\rm int} \tag{1}$$

where μ is the chemical potential, and $H_{\rm int}$

$$H_{\rm int} = \sum_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}'_1\mathbf{p}'_2} V_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}'_1\mathbf{p}'_2} a^\dagger_{\mathbf{p}_1\uparrow} a^\dagger_{\mathbf{p}_2\downarrow} a_{\mathbf{p}'_2\downarrow} a_{\mathbf{p}'_1\uparrow} \tag{2}$$

*Electronic address: suna@physics.iitm.ac.in

We add and subtract a mean field term

$$H_{\rm mf} = \sum_{\mathbf{p}\sigma} U_{\mathbf{p}} a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} + \sum_{\mathbf{p}} \Delta_{\mathbf{p}} (a^{\dagger}_{\mathbf{p},\uparrow} a^{\dagger}_{-\mathbf{p},\downarrow} + a_{-\mathbf{p},\downarrow} a_{\mathbf{p}\uparrow}) \quad (3)$$

so that $H'_0 = H_0 - \mu N + H_{\rm mf}$ and $H'_{\int} = H_{\rm int} - H_{\rm mf}$, and as a result the non-interacting Green's function

$$G_0(k) = \begin{pmatrix} G_{11}(k) & G_{12}(k) \\ G_{21}(k) & G_{22}(k) \end{pmatrix}, \quad (4)$$

$$G_{\alpha\beta}(k) = \frac{A_{\alpha\beta}(\mathbf{k})}{k_0 - E_{\mathbf{k}} + i\eta} + \frac{B_{\alpha\beta}(\mathbf{k})}{k_0 + E_{\mathbf{k}} - i\eta}, \quad (5)$$

$$A(\mathbf{k}) = \begin{pmatrix} u^2_{\mathbf{k}} & u_{\mathbf{k}} v_{\mathbf{k}} \\ u_{\mathbf{k}} v_{\mathbf{k}} & v^2_{\mathbf{k}} \end{pmatrix}, \quad (6)$$

$$B(\mathbf{k}) = \begin{pmatrix} v^2_{\mathbf{k}} & -u_{\mathbf{k}} v_{\mathbf{k}} \\ -u_{\mathbf{k}} v_{\mathbf{k}} & u^2_{\mathbf{k}} \end{pmatrix}, \quad (7)$$

where $k = (k_0, \mathbf{k})$. Using the two-component field operators defined as

$$\Psi_{\mathbf{k}} = \begin{pmatrix} \Psi_{\mathbf{k}1} \\ \Psi_{\mathbf{k}2} \end{pmatrix} = \begin{pmatrix} a_{\mathbf{k}\uparrow} \\ a^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix}, \quad (8)$$

the interaction term becomes

$$H_{\rm int} = - \sum_{\mathbf{p}_1 \mathbf{p}_2 \mathbf{p}'_1 \mathbf{p}'_2} V_{\mathbf{p}_1\ -\mathbf{p}'_2\ \mathbf{p}'_1\ -\mathbf{p}_2} :\Psi^{\dagger}_{\mathbf{p}_1 1} \Psi^{\dagger}_{\mathbf{p}_2 2} \Psi_{\mathbf{p}'_2 2} \Psi_{\mathbf{p}'_1 1}:. \quad (9)$$

The self energy diagrams at second order (the first order contribution is exactly compensated by the mean field contribution), shown in Fig. 1, contain both the normal and the anomalous contributions.

2. Results

Fig. 2 shows the HFB self-energy (normal (a) and (b) and anomalous (c) and (d)) as a function of $\lambda/k_{\rm F}$ for $1/(k_{\rm F}a) = -5$ and -1 (solid red lines), the second order contribution (solid green lines) and total contribution upto second order. The normal self-energy shows a plateau for $\lambda/k_{\rm F} < 2$ in the weak-coupling limit $1/(k_{\rm F}a) = -5$(2(a)), but as the strength

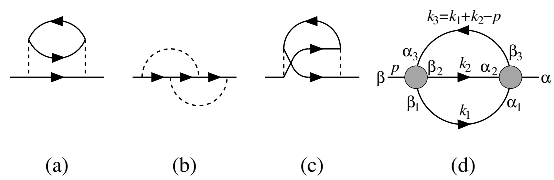

FIG. 1: Second order contribution to self energy that includes both the normal and the anomalous terms. (a) is the direct contribution, while (b) and (c) are different ways of drawing the exchange contribution. (d)represents both the direct and exchange.

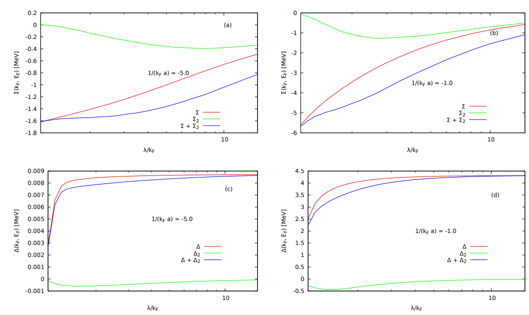

FIG. 2: Normal (a) and (b) and anomalous self-energy (c) and (d) corrections.

of the coupling increases, the independence with respect to λ disappears (2(b)). However, the anomalous self-energy while independent of λ at large values (2(c) and 2(d)) exhibits strong dependence at lower values.

Therefore, in order to understand the source of the λ depence in the self-energy, the next order in perturbation theory has to be included, which is currently the work in progress.

Acknowledgments

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References

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