Transport Coefficients of Neutron Star Matter

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Introduction

The study of the physics and astrophysics of neutron stars [1–3] provides an unique opportunity to analyze matter under extreme conditions. The recent detection of gravitational waves from binary neutron star mergers by the LIGO-Virgo collaboration offers a new context for examining the properties of super-dense, strongly interacting matter. In the post-merger remnant, there are significant density oscillations [4] that can generate observable gravitational waves. The oscillations and dissipative processes in post-merger neutron stars (NSs) are modeled using relativistic hydrodynamics, with the rate of dissipation being governed by the transport coefficients.

Formalism

We consider the RMF model for the description of hadronic matter. In this model, the Lagrangian is given by:

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}(i\gamma_\mu\partial^\mu - m_N)\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\
&- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho_\mu}\vec{\rho^\mu} \\
&+ (g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi - \frac{1}{2}g_\rho\bar{\psi}\gamma_\mu\vec{\tau}.\vec{\rho^\mu}) \\
&- \frac{1}{3}bm(g_\sigma\sigma)^3 - \frac{c}{4}(g_\sigma\sigma)^4 + \Lambda_\omega g_\omega^2(\omega_\mu\omega^\mu)(g_\rho^2\vec{\rho_\mu}\vec{\rho^\mu}) \\
&+ \sum_{l=e,\mu}\bar{\psi}_l(i\gamma_\mu\partial^\mu - m_l)\psi_l \ . \qquad (1)
\end{aligned}
$$

To simplify the solution to the field equations, we consider the mean field approximation. The Lagrangian after mean field approximation reads as

$$
\begin{aligned}
\mathcal{L}_{RMF} &= -\frac{1}{2}m_\sigma^2\bar{\sigma}^2 + \frac{1}{2}m_\omega^2(\bar{\omega}^0)^2 + \frac{1}{2}m_\rho^2(\bar{\rho}_3^0)^2 \\
&+ \Lambda_\omega(g_\omega\bar{\omega}^0)^2(g_\rho\bar{\rho}_3^0)^2 - \frac{1}{3}bm\big(g_\sigma\bar{\sigma}\big)^3 \\
&- \frac{c}{4}\big(g_\sigma\bar{\sigma}\big)^4. \qquad (2)
\end{aligned}
$$

The energy-momentum tensor is expressed as

$$T^{\mu\nu} = g^{\mu\nu}U + \sum_a \gamma_a \int d\Gamma_a^* \frac{p_a^\mu p_a^{*\nu}}{E_a^*} f_a. \qquad (3)$$

Where $U = -\mathcal{L}_{RMF}$ The baryon current is given by

$$J_B^\mu = \sum_a \gamma_a \int d\Gamma_a^* \frac{p_a^{*\mu}}{E_a^*} f_a. \qquad (4)$$

The general form of the irreversible part of the energy-momentum tensor ($\Delta T^{\mu\nu}$) and baryon current (ΔJ_B^μ) under Landau definition of four-velocity u^μ is

$$
\begin{aligned}
\Delta T^{\mu\nu} &= \eta\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu + \frac{2}{3}\Delta^{\mu\nu}\partial\cdot u\right) \\
&- \zeta\Delta^{\mu\nu}\partial\cdot u \ , \qquad (5) \\
\Delta J_B^\mu &= \lambda\left(\frac{n_B T}{w}\right)^2 \nabla^\mu\left(\frac{\mu_B}{T}\right) \ . \qquad (6)
\end{aligned}
$$

The transport coefficients η and λ mentioned in the above equation calculated using the relaxation time approximation at very low temperature (leading to $\frac{f^0(1-f^0)}{T} \approx \delta(E^* - \mu^*)$) are given by:

$$
\begin{aligned}
\eta &= \frac{1}{15}\sum_a \gamma_a \int d\Gamma_a^* \frac{|p_a^*|^4}{E_a^{*2}}\tau_a(E_a^*)\delta\left(E_a^* - \mu_a^*\right), \qquad (7) \\
\lambda &= \frac{1}{3T}\left(\frac{w}{\rho_B}\right)^2 \sum_a b_a\gamma_a \int d\Gamma_a^* \frac{|p_a^*|^2}{E_a^{*2}}\tau_a(E_a^*) \\
&\quad \left(1 - \frac{\rho_B\mu_a}{w}\right)^2 \delta\left(E_a^* - \mu_a^*\right) \ . \qquad (8)
\end{aligned}
$$

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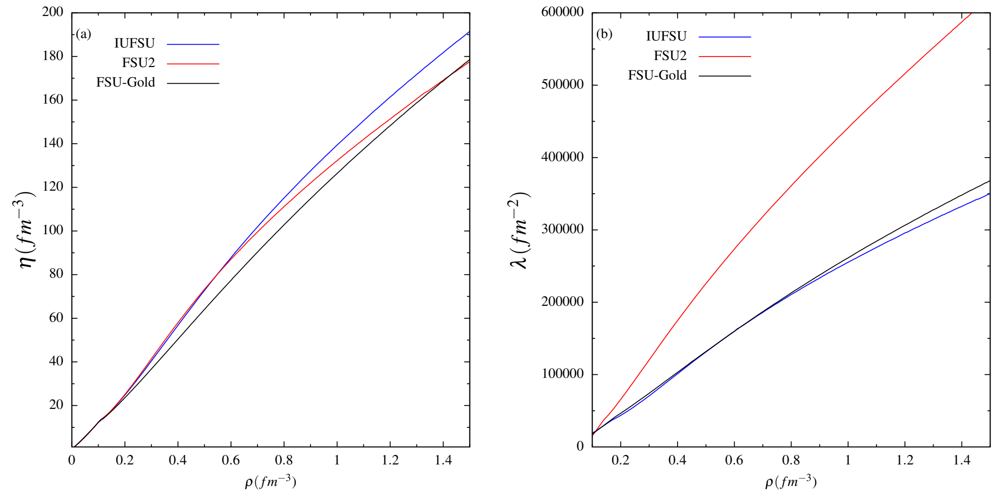

FIG. 1: *Variation of shear-viscosity (a) and bulk-viscosity (b) with density for different hadronic models.*

The relaxation time for each species of particle undergoing 2-to-2, 2-to-1 and 1-to-2 processes is given by:

$$\begin{aligned}\frac{1}{\tau_a(\mu_a^*)} &= \sum_{bcd}\frac{\gamma_b T}{1+\delta_{ab}}\frac{\sqrt{(\mu_b^*)^2-(m_b^*)^2}}{(2\pi)^2\,\mu_k^*} \\ \int d(\cos\Theta) & \quad \frac{\sqrt{\lambda\left(s,(m_a^*)^2,(m_b^*)^2\right)}}{2}\sigma_{a,b\to c,d}(s) \\ &+ \sum_{cd}\frac{m_a^*}{2\mu_a^*}\Gamma_{a\to c,d} \\ &+ \sum_{bc}\frac{\sqrt{(p_f)^2+(m_a^*)^2}}{2\mu_a^*}\Gamma_{c\to a,b} \quad . \qquad (9)\end{aligned}$$

The interactions between the constituent particles are described by the cross-section. For the nucleon-nucleon cross-section, we have used a parameterized form of the vacuum cross-section, while for leptons, we have employed a field-theoretic approach.

Results

We have calculated the shear viscosity η and thermal conductivity λ for three established variants of the RMF models: IUFSU [5], FSU2 [6], and FSUGold [7]. In Fig. (1) we have studied the variation of transport coefficient with baryon density ρ. We find that the transport coefficients increases with the baryonic density. We also found that the transport coefficients are more sensitive to electron density compared to density of other particles.

References

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