Helium Abundance Constraint on Black Hole Formation

S. Swain[1,2],[*] D. Sahu[2], G. Sahoo[1], and B. Nayak[2]
[1]*Department of Physics and Astronomical Science,*
Central University of Himachal Pradesh,
Dharamshala - 176206, Himachal Pradesh, India and
[2]*P. G. Department of Physics, Fakir Mohan University, Balasore - 756019, Odisha, India*

Introduction

Big-Bang Nucleosynthesis (BBN) is an observational milestone of the hot Big-Bang (BB) cosmology and is one of the best phenomenon to understand the origin of the light elements. Helium is the second most abundant element in the visible universe, hence influenced by theory of its abundance from primordial nucleosynthesis. In this work, we have discussed the constraints on Black Holes (BHs) derived from measurements of the Helium abundances. Here we examine a particular kind of nucleosynthesis constraint in standard cosmology i.e. the constraint on the increase in production of Helium-4 due to the injection of BH hardons by BHs [1]. We are interested in the evaluation of initial mass fraction of BH which indicates the relative density of BHs compared to cosmological density.

Basic Formalism

The complete evolution equation of BH can be written as [2]

$$\dot{M}_{\rm evo} = -\frac{\sigma_H}{256\pi^3}\frac{1}{G^2M^2} + 4\pi f R_{\rm BH}^2\rho, \qquad (1)$$

where σ_H is the Stephan-Boltzmann constant, f represents the accretion efficiency, ρ is the density of surrounding energy-matter and $R_{\rm BH}$ indicates the radius of the black hole. Here, the first term in the R.H.S. is due to Hawking evaporation and second term of R.H.S. is due to the accumulation of energy-matter from the surroundings, termed as accretion. Again, the expression for total number density of released particles from the whole evaporation process of BHs of some initial mass can be represented as [3]

$$N_{\rm em} = \frac{\rho_{\rm BH}}{< E_{\rm em} >}, \qquad (2)$$

where $< E_{\rm em} >$ represents the average energy of the emitted particles. Now, e can calculate the ratio of the energy density of BHs at evaporation time to the background radiation energy density as

$$\alpha_{\rm evp} = \frac{\rho_{\rm BH}}{\rho_{\rm rad}} = \frac{N_{\rm em} < E_{\rm em} >}{N_{\rm rad} < E_{\rm rad} >}. \qquad (3)$$

Here, the final mass fraction is denoted by α_{evp}, $< N_{rad} >$ represents the number density of the particles and $< E_{rad} >$ shows the average energy density of the particles. The average energies ratio shows the approximated ratio of background temperature at the time of evaporation to the onset of evaporation of the BH temperature

$$\frac{< E_{\rm rad} >}{< E_{\rm em} >} = \frac{T_{\rm evp}}{T_{\rm BH}}. \qquad (4)$$

Again, the standard cosmological temperature - time relation [4] is given by the formula

$$t = 0.301 g_*^{-1}\frac{m_{\rm pl}}{T^2}, \qquad (5)$$

where g_* is a constant with value 10.78. So, one can rewrite the Eq. (4), by considering the above relation and the analysis of Nayak et al.[5] as

$$\frac{< E_{\rm rad} >}{< E_{\rm em} >} = \frac{T_{\rm evp}}{T_{\rm BH}} = 7.28\left(\frac{M(t_i)}{M(t_c)}\right)\left(\frac{M(t_c)}{m_{\rm pl}}\right)^{-\frac{1}{2}}. \qquad (6)$$

---

[*] Electronic address: `ssuryakanta17@gmail.com`

During nucleosynthesis, the total emitted number density at evaporation time can be expressed as

$$N_{\rm em} = 7.28\left(\frac{M(t_i)}{M(t_c)}\right)\alpha_{\rm evp}\left(\frac{M(t_c)}{m_{\rm pl}}\right)^{-\frac{1}{2}} N_{\rm rad}. \tag{7}$$

By taking the observational point of view, Clancy et al. [3] examined that

$$N_{\rm em} < \left(\frac{2.8}{100F}\right)n_b, \tag{8}$$

where F denotes the fraction of total number of particle emitted by the BH and its value is $F \leq 0.2$. Now, by comparing both the Eq. (7) and Eq. (8), one can get

$$\alpha_{\rm evp} < \frac{0.38}{F}\times 10^{-2}\left(\frac{M(t_c)}{M(t_i)}\right)\left(\frac{M(t_c)}{m_{\rm pl}}\right)^{\frac{1}{2}}\eta_{\rm evp}, \tag{9}$$

where $\eta_{\rm evp}$ shows the baryon to photon ratio at the time of evaporation. By considering the relation $\eta_{\rm evp} = \eta_0$, where $\eta_0 \approx 2.8\times 10^{-8}\Omega_b h^2$ ($\Omega_b h^2 \approx 0.02$), the Eq. (9) reduces to

$$\alpha_{\rm evp} < 1.064\times 10^{-11}\left(\frac{M(t_c)}{M(t_i)}\right)\left(\frac{M(t_c)}{m_{\rm pl}}\right)^{\frac{1}{2}}. \tag{10}$$

The general expression for $\alpha_{\rm evp}$ an be written as [5]

$$\alpha_{\rm evp} = \alpha_i\left(\frac{M(t_c)}{M(t_i)}\right)\left(\frac{a(t_{\rm evp})}{a(t_i)}\right), \tag{11}$$

where $M(t_c)$ represents the maximum accumulation of energy-matter by black holes. After comparing both the Eq. (10) and Eq. (11), we get

$$\alpha_i < 1.064\times 10^{-11}\left(\frac{a(t_i)}{a(t_{\rm evp})}\right)\left(\frac{M(t_c)}{m_{\rm pl}}\right)^{\frac{1}{2}}. \tag{12}$$

Results and Discussion

One can see from TABLE I and FIG. 1 that due to inclusion of accretion, the constraint coming from Helium abundance on the initial mass fraction of the BHs becomes more stronger. Further the upper bounds on the initial mass fraction of the black holes are found to be more stringent than that of Brans-Dicke theory[5].

TABLE I: Calculation of initial mass fraction of Black Holes (α_i) for various accretion efficiencies (f).

Helium Abundance Constraint	
Accretion efficiency (f)	Initial mass fraction $\alpha_i <$
0	2.6086×10^{-19}
0.05	2.5089×10^{-19}
0.15	2.2965×10^{-19}
0.25	2.0623×10^{-19}
0.35	1.7978×10^{-19}
0.45	1.4871×10^{-19}
0.55	1.0913×10^{-19}
0.65	4.1246×10^{-20}

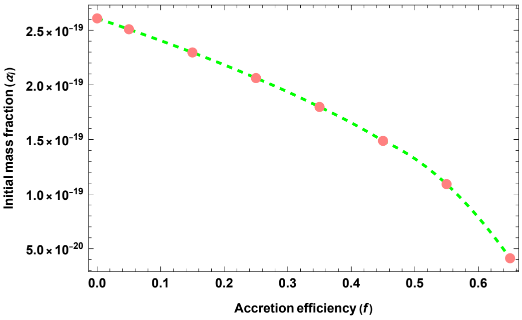

FIG. 1: Variation of initial mass fraction of Black Holes (α_i) with accretion efficiencies (f) due to Helium abundance.

References

[1] K. Kohari and J. Yokoyama, Phys. Rev. D **61**, 023501 (2000).

[2] S. Swain, G. Sahoo and B. Nayak, Sci Rep **14**, 16928 (2024).

[3] D. Clancy, R. Guedens and A. R. Liddle, Phys. Rev. D **68**, 023507 (2003).

[4] E. W. Kolb and M. S. Turner, The Early Universe, Addison-Wesley, New York, U.S.A. (1990).

[5] B. Nayak, A. S. Majumdar and L. P. Singh, J. Cosmol. Astropart. Phys. **2010**, 039 (2010).