Model-Independent Equation of State from Neutron Star Observables

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Introduction

Neutron stars (NSs) are incredibly dense objects, packing more mass than the Sun into a radius of about 10-12 km. They offer a rare opportunity to study matter at extreme densities, far beyond those in atomic nuclei. Research on NSs advances our understanding of dense matter, gravitational waves, and stellar evolution. Collaborative efforts continue to push scientific boundaries, making NSs a key to unlock new insights into the universe.

The macroscopic properties of neutron stars, such as mass, radius, and tidal deformability, are significantly influenced by the nuclear microphysics of their interiors. However, certain combinations of these properties are surprisingly insensitive to the internal structure. In hydrostatic equilibrium, the equation of state (EOS) dictates the mass-radius (M-R) relation through the Tolman-Oppenheimer-Volkoff (TOV) equations, where the mapping between $P-\rho$ to the M-R relation is a bijection. Thus, it is possible to reconstruct the EOS from the M-R relation. In this work, we construct the EOS in a model-independent manner using NS properties obtained by diverse sets of EOSs as briefly described in the following section.

Collections of EOSs

We have used seven diverse sets of EOSs in the present study, namely, NL, NL-hyp, CompOSE, CSE, PWP, DDB, and SR models [1, 2]. The **NL** set correspond to the β-equilibrium matter using the relativistic mean field (RMF) framework, while **NL-hyp** includes hyperons with similar constraints. The **CompOSE** set comprises general-purpose cold EOSs from the CompOSE database, and the **CSE** set uses an agnostic approach with constraints from nuclear and astrophysical data. The **PWP** set applies a piecewise-polytrope model, and the **DDB** set uses the RMF framework with density-dependent couplings. The **SR** set employs spectral representation for EOS construction, and the **All** set includes 100 sampled EOSs from each of the seven sets.

Results

We employ the above collection of EOSs to build a database containing various neutron star properties like mass (M), radius (R), and tidal deformability (Λ), obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations. The density (ρ_c, energy density (ϵ_c) and pressure (P_c) at the centre of a neutron star are mapped to the corresponding mass and radius within the symbolic regression method (SRM). In the symbolic regression method, our target **Y** vector comprise of the ρ_c, ϵ_c, and P_c for each EOSs and the corresponding mass and radius are stored in feature vector **X**. The optimal equations are,

$$\frac{\rho_c}{\text{fm}^{-3}} = M\left(-0.516 + \frac{10.160}{R}\right) \quad (1)$$

$$\frac{\epsilon_c}{\text{MeV.fm}^{-3}} = M\left(-704.222 + \frac{12691.952}{R}\right) \quad (2)$$

$$\frac{P_c}{\text{MeV.fm}^{-3}} = \frac{105.623\, M^2}{(-M + R - 7.753)} \quad (3)$$

In Table-I, we present Pearson's correlation coefficients $'r'$ between the actual and predicted values of ρ_c, ϵ_c, and P_c, along with

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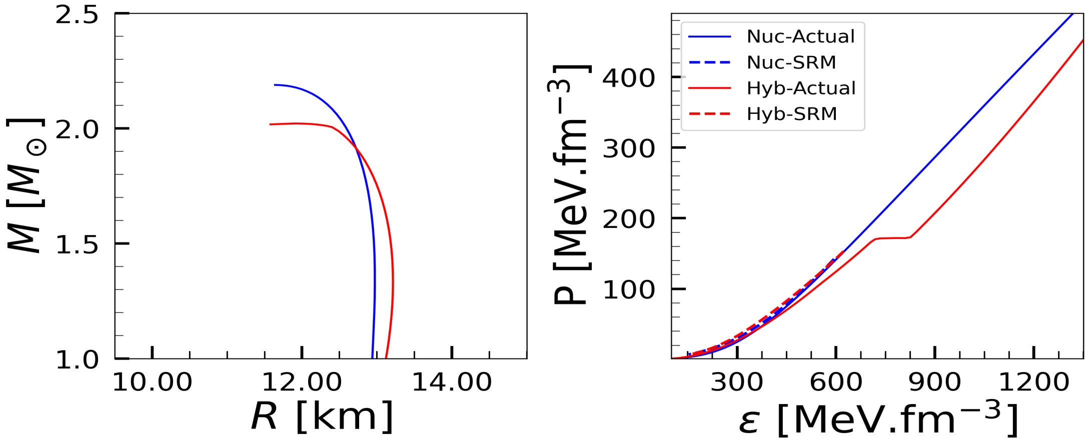

FIG. 1: *Left panel:* M-R curve for nucleonic (Nuc) and hybrid (Hyb) EOSs. *Right panel:*The pressure as a function of the energy density. The dashed lines are obtained using the Eqs. (2&3).

TABLE I: The correlation coefficients ($'r'$) and relative error (RE) from Eq. (1) - (3).

Y	**Eqs.**		**Datasets**							
			NL	**NL-hyp**	**CompOSE**	**CSE**	**PWP**	**DDB**	**SR**	**All**
$\frac{\rho_c}{\mathrm{fm}^{-3}}$	1	$'r'$	0.98	0.97	0.95	0.92	0.94	0.97	0.97	0.94
		RE (%)	6.8	5.2	8.3	10.5	9.1	6.2	5.9	7.5
$\frac{\epsilon_c}{\mathrm{MeV.fm}^{-3}}$	2	$'r'$	0.98	0.97	0.94	0.93	0.94	0.97	0.97	0.93
		RE (%)	5.5	6.1	8.9	13.6	12.2	5.4	5.7	8.3
$\frac{P_c}{\mathrm{MeV.fm}^{-3}}$	3	$'r'$	0.99	0.97	0.98	0.99	0.99	0.99	0.99	0.98
		RE (%)	5.8	11.1	11.4	12.9	11.6	6.5	8.6	9.7

the relative error (RE) for different EOS sets. The higher values of correlation coefficients and smaller values of RE in the last column obtained for the case 'All' clearly emphasize the model independence of Eqs.(1-3).

We use these Eqs. to construct EOSs for a randomly selected M-R relations for nucleonic matter and for the hybrid matter. In Figure-1 we show the M-R curves (left) and the corresponding actual and predicted EOSs from Eqs. (1)- (3) (right). The nucleonic EOS is accurately predicted up to energy densities corresponding to a neutron star mass of 2 $M_\odot$, while the hybrid EOS prediction deviates only by about 10% at higher energy densities. Similar trends are observed for the EOS obtained from the mass - tidal deformability relation (not shown).

Conclusion

We have applied symbolic regression method to obtain model independent relations between the neutron star properties and central EOS parameters. These relations are employed to construct EOSs from mass - radius relations for different compositions. The model independent EOSs so obtained are found to agree remarkably well with the actual ones. Our predictions are more accurate for the EOS related to nucleonic matter. In the case of hybrid matter, the predicted EOS differs only by less than 10% in the high-density region.

References

[1] S. Typel *et al.*, [arXiv:1307.5715] (2015)
[2] T. Malik *et al.*, [arXiv:2301.08169] (2023)