Neutron star properties within energy-momentum squared gravity

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1. Introduction

Precise understanding of the equation of state at supranormal densities in compact astrophysical objects such as neutron stars rely on the accurate knowledge of the nuclear and gravitational interactions [1]. The repulsive nuclear equation of state and the balancing attractive strong-field gravitational physics are interlinked by the Tolman-Oppenheimer-Volkoff (TOV) equations for the hydrostatic equilibrium of the star configuration. While theoretical calculations combined with laboratory experiments can provide nuclear matter properties only about the saturation density, information at high density can be obtained from astrophysical observations.

Since neutron stars are superdense objects with strong gravitational fields, they provide direct information about physics in the strong field domain. Therefore, in addition to the conventional method based on general relativity (GR), it will be interesting to investigate alternate theories of gravity in superdense stars. We have calculated the neutron star properties for Relativistic Mean Field (RMF) and Skyrme-Hartree-Fock (SHF) equation of state within the framework of GR as well as energy-momentum squared gravity (EMSG).

2. Formalism

In the EMSG theory, the Einstein-Hilbert action is modified by the addition of a scalar term $\alpha T_{\mu\nu}T^{\mu\nu}$ leading to

$$S = \int \left[\frac{1}{2\kappa}\left(\mathcal{R} - 2\Lambda\right) + \alpha T_{\mu\nu}T^{\mu\nu} + \mathcal{L}_{\mathrm{m}}\right]\sqrt{-g}\,\mathrm{d}^4x, \quad (1)$$

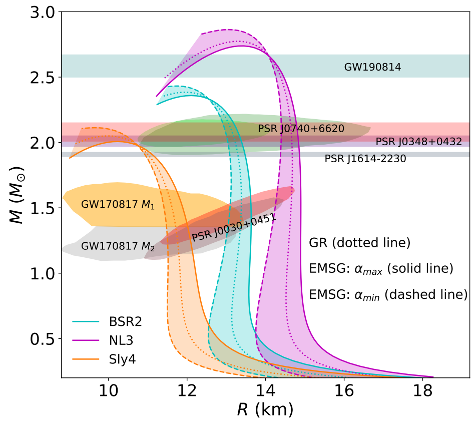

FIG. 1: Mass-radius curves for neutron stars in the nuclear EoSs BSR2, NL3, and Sly4 in GR and EMSG theories.

in the usual notation [2, 3]. The Einstein's field equation for the EMSG action is

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} + \kappa\alpha\left(g_{\mu\nu}T_{\sigma\epsilon}T^{\sigma\epsilon} - 2\theta_{\mu\nu}\right), \quad (2)$$

where $G_{\mu\nu}$ is the Einstein tensor, and the expression for the tensor $\theta_{\mu\nu}$ can be found in Ref. [2, 3]. The EMSG theory provides non-minimal matter and geometry coupling and introduces a higher-order contribution to the material stresses on the right-hand side of the Einstein equation. The field equations of EMSG with the conventional physical energy-momentum tensor could be mapped into the GR Einstein's field equations with an effective or modified energy-momentum tensor. This enables the straightforward calculation of the stellar properties. The modified TOV equation can be derived by solving the EMSG field equations for a spherically symmetric body consisting of perfect fluid matter.

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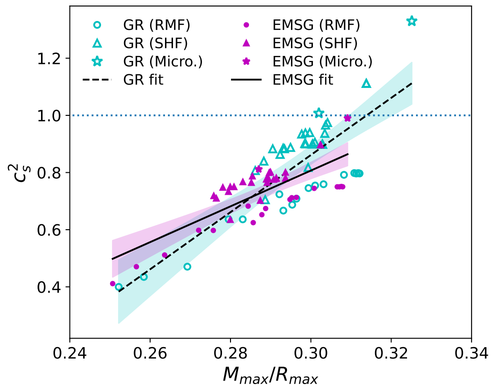

FIG. 2: The central speed of sound squared c_s^2 as a function of the compactness $M_{\rm max}/R_{\rm max}$ corresponding to maximum mass stars.

3. Results and Discussions

The mass-radius relations obtained within EMSG are shown in Fig. 1 for three different EoSs: Sly4, BSR2 and NL3. The contours and bands represent mass-radius constraints derived from NICER observations and gravitational wave data [4, 5]. The maximum value of the EMSG parameter $\alpha_{\rm max} \approx 10^{-37}$ cm^3/erg, is taken from the Ref. [3]. The minimum value is determined from the stability criteria of the star. Compared to GR, the EMSG model results in an effective softening of all the EoSs when $\alpha > 0$, and an effective stiffening of all the EoSs when $\alpha < 0$. The maximum mass calculated within EMSG remains nearly unchanged in comparison to GR prediction. It can be noticed that the radius of a neutron star increases for positive values of α and decreases for negative values of α. This indicates that a positive value of α weakens the gravitational interaction, leading to an increase in the radius, while a negative value of α strengthens the interaction, resulting in a decrease in the radius. The radius of a $1M_\odot$ star deviates from the GR estimate by about 0.6 km for the upper limit of the EMSG parameter α. The softest Sly4 has the highest increase in the radii ΔR for a constant positive value of α, whereas the stiffest NL3 exhibits the maximum decrease in ΔR for a constant positive value of α.

The relations between the speed of sound squared c_s^2 and the compactness $M_{\rm max}/R_{\rm max}$ corresponding to maximum mass stars in the RMF, SHF and microscopic models of EoS are displayed in Fig. 2. It has been found that the compactness of the NSs increases the central speed of sound. The EMSG theory, which predicts slightly larger $R_{\rm max}$, has a smaller sound speed compared to GR. In EMSG theory, the causality condition imposes an upper limit on the compactness, $C_{\rm max} \equiv M_{\rm max}/R_{\rm max} \lesssim 0.338$, which in turn provides a lower bound on the radius, $R_{\rm max}/{\rm km} \gtrsim 4.370 M_{\rm max}/M_\odot$. Similarly, in GR, the maximum compactness $C_{\rm max} \lesssim 0.314$ leads to a radius bound of $R_{\rm max}/{\rm km} \gtrsim 4.704 M_{\rm max}/M_\odot$. These radii lower bounds obtained from our analysis are consistent with the NICER measurements for the PSR J0740+6620, radius of about $12.39^{+1.30}_{-1.98}$ km with a mass $M \approx 2.072^{+0.067}_{-0.066} M_\odot$ and for the PSR J0030+0451, radius $\approx$ $12.71^{+1.14}_{-1.19}$ km with a mass $\approx$ $1.34^{+0.15}_{-0.16} M_\odot$ [4, 5].

4. Summary

In summary, we investigate the properties of neutron stars, particularly their mass and radius, within the framework of EMSG. The results are compared with those obtained from GR. Additionally, we analyze the correlation between the central speed of sound and the compactness of the maximum mass configuration. From this correlation, we establish lower bounds on the neutron star radius in both GR and EMSG. Our results are consistent with recent astrophysical observations.

References

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