Wigner Distributions associated to T-odd $H^o_{1,4}$ and $H^o_{1,7}$ in longitudinal position space

Tanmay Maji[1]*
[1]*National Institute of Technology Kurukshetra, Haryana - 136119, INDIA*

Introduction

Wigner distributions provide a comprehensive picture of the proton's partonic structure by combining spatial and momentum information. They are derived from the Fourier transform of GTMDs with respect to the transverse momentum transfer. In Quantum Chromodynamics (QCD), Wigner distributions have been widely studied in the transverse impact parameter space. Recent work has explored Wigner distributions in the boost-invariant longitudinal position space, denoted as σ, related to the skewness parameter ζ. These distributions reveal diffraction patterns similar to wave optics and exhibit long-distance tails.

The boost-invariant longitudinal position variable, σ, which is the Fourier conjugate to skewness ζ, reveals longitudinal impact parameter details within the proton. This work focuses on the investigation of Wigner distribution in boost invariant longitudinal position space which involves the leading-twist T-odd (time reversal odd) part of GTMDs $H_{1,4}$ and $H_{1,7}$ with non-zero skewness. At the TMD limit $\Delta = 0$, these two GTMDs $H_{1,4}$ and $H_{1,7}$ reduce to $h^{\perp}_{1T}$ and $h^{\perp}_{1L}$ respectively. $h^{\perp}_{1T}$ represents the distribution corresponding to a transversely polarised quark in a transversely polarised proton at a mutually perpendicular direction, whereas, $h^{\perp}_{1L}$ is the distribution for a transversely polarised quark in a longitudinally polarised proton.

We consider deep inelastic scattering of a proton of average momentum P interacts with lepton via a photon of virtuality $Q^2 = -q^2$. Eventually, the high energy virtual photon interacts with the interior parton of momentum p which caries longitudinal momentum fraction $x = p^+/P^+$ and transverse momentum $\mathbf{p}_\perp$. The momentum transfer to the system is defined by Δ and $-t = \Delta^2$. The Fourier conjugate to the skewness ξ is $\sigma = \frac{1}{2}b^- P^+$ and the WDs in boost invariant longitudinal position space are defined as

$$\tilde{\rho}^{\nu}_{1,4}(x,\sigma,t,\mathbf{p}_\perp) = -\int_0^{\xi_s} \frac{d\xi}{2\pi} e^{i\sigma.\xi} \sqrt{1-\xi^2} \times H^{o\nu}_{1,4}(x,\xi,t,\mathbf{p}^2_\perp,\mathbf{\Delta}_\perp.\mathbf{p}_\perp), \quad (1)$$

$$\tilde{\rho}^{\nu}_{1,7}(x,\sigma,t,\mathbf{p}_\perp) = -\int_0^{\xi_s} \frac{d\xi}{2\pi} e^{i\sigma.\xi} \frac{1}{\sqrt{1-\xi^2}} \times H^{o\nu}_{1,7}(x,\xi,t,\mathbf{p}^2_\perp,\mathbf{\Delta}_\perp.\mathbf{p}_\perp), \quad (2)$$

In this Fourier transformation, finiteness of the upper-limit ξ_s is restricted by the allowed range given by ξ_{max} as

$$\xi_{max} = \frac{(-t)}{2M^2}\left(\sqrt{1+\frac{4M^2}{(-t)}} - 1\right), \quad (3)$$

where M is the mass of target proton [1–3]. We restrict ourself to the DGLAP region $\xi < x < 1$ and the limit of the Fourier transform in σ-space is taken as $\xi_s = \xi_{max}$ if $\xi_{max} < x$ and $\xi_s = x$ for $\xi_{max} > x$, determined by a fixed value of $-t$.

We use the Light-Front Quark Diquark Model (LFQDM) within the DGLAP region. The model includes final-state interactions (FSI) and utilizes the AdS/QCD soft-wall model to construct wave functions [4]. We present analytical and numerical results for T-odd GTMDs, explore quark spin density, and examine Wigner distributions in boost invariant longitudinal position space.

Model Results

Here we present the LFQDM results of T-odd part of $H^{(o)\nu}_{1,4}$ and $H^{(o)\nu}_{1,7}$. The model parameters are taken from Ref.[4] with proton mass $M = 0.94\ GeV$, diquark mass $m_D = 0.98 \pm 0.04\ GeV$. In principle, the mass of diquark has a lower bound given by the proton mass to satisfy the condition of stable bound state proton. However, the upper bound of m_D is restricted by the spin sum rule [5]. The quarks are considered to be

*Electronic address: tanmayphy@nitkkr.ac.in

massless.

$$H_{1,4}^{(o)\nu} = -N_{H14}^{\nu}\frac{1}{16\pi^3}\frac{1}{\sqrt{1-\xi^2}}\left(\chi_2'-\chi_2''\right) \quad (4)$$

$$\times\frac{2}{x''x'}A_2^{\nu}(x'')A_2^{\nu}(x')\exp[J]\,, \quad (5)$$

$$H_{1,7}^{(o)\nu} = -N_{H17}^{\nu}\frac{1}{16\pi^3}\sqrt{1-\xi^2}\frac{1}{2}\bigg[\left(\chi_2'-\chi_1''\right) \times\frac{1}{x'}A_1^{\nu}(x'')A_2^{\nu}(x') - \left(\chi_1'-\chi_2''\right) \frac{1}{x''}A_2^{\nu}(x'')A_1^{\nu}(x')\bigg]\exp[J]\,, \quad (6)$$

The normalization constants $N_{\Lambda\lambda}^{\nu}$ are

$$N_{H17}^{\nu} = \left(C_S^2N_S^2 + C_A^2\big(\frac{1}{3}N_0^2 - \frac{2}{3}N_1^2\big)\right)^{\nu},$$

$$N_{H14}^{\nu}\,, = \left(C_S^2N_S^2 - C_A^2\frac{1}{3}N_0^2\right)^{\nu}, \quad (7)$$

Where the GTMDs $H_{1,4}^{(o)\nu}$ and $H_{1,7}^{(o)\nu}$ are function of $(x,\xi,\mathbf{\Delta}_{\perp}^2,\mathbf{p}_{\perp}^2,\mathbf{\Delta}_{\perp}.\mathbf{p}_{\perp})$ and $\exp[J] = \exp\big[-\tilde{a}(x'')\mathbf{p}_{\perp}''^2 - \tilde{a}(x')\mathbf{p}_{\perp}'^2\big]$.

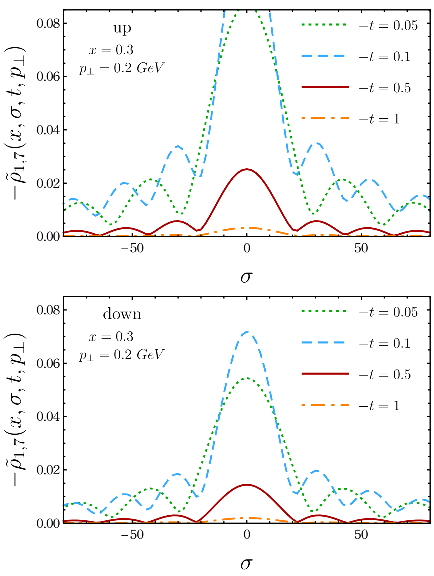

FIG. 1: Model results of $\tilde{\rho}_{1,7}^{\nu}(x,\sigma,t,\mathbf{p}_{\perp})$ for u and d-quarks in boost invariant longitudinal space.

Fig.1 presents LFQDM results of the Wigner distributions $\tilde{\rho}_{1,7}^{\nu}(x,\sigma,t,\mathbf{p}_{\perp})$ for u and d-quarks. The different plots are for different values of energy transfer to the system $-t = 0.05, 0.1, 0.5, 1$. The distributions in boost invariant space show oscillatory behavior which is analogous to optical phenomenon by diffraction gratings. Here, the energy transfer $-t$ can be considered to be similar to the slit, and the first maxima varies inversely.

Conclusion

We examine the longitudinal impact parameter configuration of Wigner Distributions involving T-odd leading twist GTMDs when the constituent quark is transversely polarised in a longitudinally and transversely polarised proton. The Wigner Distributions in boost invariant longitudinal parameters space associated with $H_{1,4}^{o}$ and $H_{1,7}^{o}$ shows diffraction behavior analogous to the Optics phenomenon.

Acknowledgments

This work is supported by the Science and Engineering Research Board (SERB) through the SRG (Start-up Research Grant) of File No. SRG/2023/001093.

References

[1] S. J. Brodsky, M. Diehl and D. S. Hwang, Nucl. Phys. B **596**, 99-124 (2001) doi:10.1016/S0550-3213(00)00695-7 [arXiv:hep-ph/0009254 [hep-ph]].

[2] D. Chakrabarti, R. Manohar and A. Mukherjee, Phys. Rev. D **79**, 034006 (2009) doi:10.1103/PhysRevD.79.034006 [arXiv:0811.0521 [hep-ph]].

[3] R. Manohar, A. Mukherjee and D. Chakrabarti, Phys. Rev. D **83**, 014004 (2011) doi:10.1103/PhysRevD.83.014004 [arXiv:1012.2627 [hep-ph]].

[4] T. Maji and D. Chakrabarti, Phys. Rev. D **94**, no.9, 094020 (2016) doi:10.1103/PhysRevD.94.094020 [arXiv:1608.07776 [hep-ph]].

[5] D. Chakrabarti, P. Choudhary, B. Gurjar, T. Maji, C. Mondal and A. Mukherjee, Phys. Rev. D **109**, no.11, 114040 (2024) doi:10.1103/PhysRevD.109.114040 [arXiv:2402.16503 [hep-ph]].