Investigating the mass spectra of all strange tetraquark

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Introduction

As a result of the success of the quark model, numerous unconventional or exotic bound states were hypothesized in the 1980s. Over the subsequent decades, these unconventional states were theoretically described in various forms, including hadronic molecules, hybrid mesons, tetraquarks, and pentaquarks [1–4]. During this period, a significant number of resonances were detected, exhibiting properties consistent with these exotic states. Specifically, four-quark resonances are typically classified as tetraquarks, with the first candidate for such a state being observed in 2003[5]. So far more than 15 candidates for tetraquark state have experimentally been observed and most of them consist of atleast one charm quark resonance. The unexpected finding of tetraquark with all charm in 2020 inspired researchers to explore for its bottom and strange counterparts [6].

The objective of this article is to investigate the mass spectra of all-strange tetraquark states $[ss][\bar{s}\bar{s}]$, inspired by our previous studies [7–12].

Theoretical Framework

A compact tetraquark state is generally described as a bound system of four quarks, consisting of a diquark $[qq]$ and an antidiquark $[\bar{q}\bar{q}]$, bound together by color forces in a color-neutral configuration. The interaction between two quarks or two antiquarks, facilitated by gluon exchange, leads to the formation of bound states known as diquarks and antidiquarks.

In our study, we employed the Coulomb-plus-quadratic potential $V_{C+L}(r)$, which includes a gluonic interaction represented by a Coulombic term, while the quark confinement is modeled using a linear term:

$$V_{C+L}(r) = \frac{k_s \alpha_s}{r} + br + c$$

Here, k_s and α_s represent the strong coupling constant and the confinement coefficient, respectively. Additionally, we incorporate a non-perturbative relativistic mass correction through the term $V^1(r)$, based on the formalism outlined in Refs. [7], given by:

$$V^1(r) = -\frac{C_F C_A}{4}\frac{\alpha_s^2}{r^2}$$

where C_F and C_A are the Casimir operators for the fundamental and adjoint representations, respectively.

To account for the spin-dependent interactions, we include perturbative corrections to the central potential as described in Ref. [7], with the spin-dependent potential $V_{SD}(r)$ given by:

$$V_{SD}(r) = V_{SS}(r) + V_{LS}(r) + V_T(r)$$

The fine structure of the state is influenced by the spin-orbit interaction term $V_{LS}(r)$ and the tensor term $V_T(r)$, while the spin-spin interaction term $V_{SS}(r)$ is responsible for hyperfine splitting [13].

Using this potential framework, we calculated the mass spectra of diquark $[ss]$, antidiquark $[\bar{s}\bar{s}]$, and tetraquark states T_{4s}. The diquark and antidiquark masses are given by:

$$M_{(ss)} = 2M_s + E_{(ss)} + \langle V^1(r) \rangle$$

where, m_s and $E_{(ss)}$ are the constituent strange quark mass and the binding energy

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of the diquark respectively. Finally, the mass of the tetraquark state $ss\bar{s}\bar{s}$ is calculated as:

$$M_{ss\bar{s}\bar{s}} = m_{ss} + m_{\bar{s}\bar{s}} + E_{[ss][\bar{s}\bar{s}]} + \langle V^1(r)\rangle$$

This framework provides a robust theoretical approach to investigate the mass spectra and structure of tetraquark states.

Results and Discussion

In this study, the mass spectra of both S-wave and P-wave all-strange tetraquark states, denoted as T_{4s}, have been calculated and compared with the corresponding two-meson thresholds M_T. The results are summarized in Table I, where the states are classified based on their quantum numbers J^{PC}, and the masses are compared with the meson thresholds and theoretical predictions. The parameters used in this analysis are taken from the most recent Particle Data Group (PDG) update [14].

TABLE I: The mass spectra of S-wave and P-wave all-strange tetraquarks along with the corresponding two-meson threshold (M_T) in MeV.

State	J^{PC}	Mass	M_T	Threshold
$^1S_0$	0^{++}	2184	1523	$\eta_s\eta_s$
$^3S_1$	1^{+-}	2248	1781	$\eta_s\phi$
$^5S_2$	2^{++}	2378	2040	$\phi\phi$
$^1P_1$	1^{--}	2799	-	-
$^3P_0$	0^{-+}	2577	2009	$\eta_s f_0$
$^3P_1$	1^{-+}	2800	2219	$\eta_s f_1$
$^3P_2$	2^{-+}	2873	2254	$\eta_s f_2$
$^5P_1$	0^{--}	2574	2198	$\eta_s h_1$
$^5P_2$	1^{--}	2855	2478	ϕf_1
$^5P_3$	3^{--}	2963	2512	ϕf_2

The mass spectra for the all-strange tetraquarks have been computed using a zeroth-order potential, incorporating both a Coulombic and a linear term. The masses of the S-wave and P-wave tetraquark states are compared with the respective two-meson thresholds.

The small deviations from the two-meson threshold, along with the comparable masses, highlight the significance of these states in expanding our understanding of multiquark systems. The computed mass spectra serve as a foundation for guiding future searches for exotic hadronic states at experimental facilities like PANDA, J-PARC, Belle, LHCb, and others which focus on in-depth analyses of resonances involving strange quarks [15–18].

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