Magnetic Moments of B_c Meson

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Introduction

Magnetic moments of hadrons are essential dynamical properties that have wide-ranging applications and can offer new insights into hadronic dynamics, while also serving as a rigorous evaluation of the dynamics incorporated in a theoretical model [1]. Various theoretical approaches have been used to examine the magnetic moments of hadrons, including the component quark model, the Bag model, the QCD sum rule, etc. [2,3]. In the present study, we utilize the Constituent Quark Model (CQM) to determine the magnetic moments of the B_c meson. The CQM conceptualizes the B_c meson as a bound state of a charm quark and anti-bottom quark, with each quark contributing to the total magnetic moment based on their respective masses and magnetic properties. Additionally, we also explore the magnetic moments of mixed states of the B_c meson.

Methodology

In the context of the constituent quark model, the magnetic moments (μ_{B_c}) of the B_c meson states and the transition magnetic moments ($\mu_{B_c \to B_c'}$) between the B_c meson states can be

calculated by evaluating the expectation values of the z-component of the total magnetic moment operator ($\hat{\mu}_z$) given by [3]

$$\mu_{B_c} = \langle J_{B_c}, J_{B_c} | \hat{\mu}_z | J_{B_c}, J_{B_c} \rangle \quad (1)$$

$$\mu_{B_c \to B_c'} = \langle J_{B_c'}, J_z | \hat{\mu}_z | J_{B_c}, J_z \rangle^{J_z = Min\{J_{B_c},\ J_{B_c'}\}} \quad (2)$$

The total magnetic moment operator is composed of spin magnetic moment and orbital magnetic moment given by

$$\hat{\mu}_z^{spin} = \sum_{j=c,\bar{b}} \mu_j \hat{\sigma}_{jz},$$

$$\hat{\mu}_z^{orbital} = \frac{m_c \mu_{\bar{b}}}{m_c + m_{\bar{b}}} \hat{L}_z + \frac{m_{\bar{b}} \mu_c}{m_c + m_{\bar{b}}} \hat{L}_z \quad (3)$$

Here $\mu_j = e_j/2m_j$. e_j, m_j and $\hat{\sigma}_{jz}$ are the charge, mass and the z-component of the Pauli spin operator of the j^{th} constituent quark, respectively. $\hat{L}_z$ is the z-component of the orbital angular momentum operator. The spin and orbital magnetic moment of a B_c state can be calculated by evaluating the expectation value of the spin and orbital magnetic moment operator using the spin-orbit and flavor wave functions. For S-wave B_c states $L_z = 0$, therefore only the spin component contributes to the magnetic moment. For the excited states there is contribution from both the spin and orbital components. The spin-orbit wave function $|^{2S+1}L_J\rangle$ for the excited B_c states can be obtained by [3]

$$|^{2S+1}L_J\rangle = \sum_{m_L, m_S} C^{JM}_{Lm_L Sm_S} Y_{L,m_L} \chi_{S,m_S} \quad (4)$$

Here $C^{JM}_{Lm_L Sm_S}$ is the Clebsch-Gordon coefficient, Y_{L,m_L} is the orbital wave function and χ_{S,m_S} is the spin wave function. The mixed states of B_c meson $|L_J\rangle$ and $|L_J'\rangle$ are related to the $L - S$ basis by [3]

$$|L_J\rangle = \cos\theta_L |^1L_J\rangle + \sin\theta_L |^3L_J\rangle$$

$$|L_J'\rangle = -\sin\theta_L |^1L_J\rangle + \cos\theta_L |^3L_J\rangle \quad (5)$$

where θ_L is the mixing angle. Then the magnetic moment of the mixed states can be calculated using the expression [3]

$$\mu_{|L_J\rangle} = \mu_{|^1L_J\rangle} \cos^2\theta_L + \mu_{|^3L_J\rangle \to |^1L_J\rangle} \sin 2\theta_L + \mu_{|^3L_J\rangle} \sin^2\theta_L$$

$$\mu_{|L_J'\rangle} = \mu_{|^1L_J\rangle} \sin^2\theta_L - \mu_{|^3L_J\rangle \to |^1L_J\rangle} \sin 2\theta_L + \mu_{|^3L_J\rangle} \cos^2\theta_L \quad (6)$$

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Results and Discussion

Applying the aforementioned methodology, the magnetic moments of B_c meson states are calculated for S, P, D, F and G, with the results presented in Table 1 and compared with those from other theoretical models. Currently there is no experimental data and limited theoretical studies for the B_c meson magnetic moments. The masses of the charm quark, $m_c = 1.319$ GeV and the bottom quark, $m_b = 4.744$ GeV are determined using a relativistic screened potential model. The states are labeled using the $\left(n^{2S+1}L_J\right)$ notation for non-mixed states and $\left(nL_J, nL'_J\right)$ mixed states. For non-mixed states, the magnetic moments are independent of the principal quantum number n, but for mixed states, they depend on n due to variations in the mixing angle. Our results are generally consistent with those of the relativistic screened model in [3], but we observe differences in the magnetic moments of mixed states, which can be attributed to variations in the mixing angles obtained in respective models.

States	Ours	[3]	[4]
n^1S_0	0	0	0
n^3S_1	0.540	0.443	
n^3P_0	0	0	0
n^3P_2	0.925	0.739	0.739
$1P_1$	0.504	0.527	
$1P'_1$	0.344	0.138	
$2P_1$	0.624	0.486	0.437
$2P'_1$	0.224	0.179	0.229
$3P_1$	0.649	0.469	0.454
$3P'_1$	0.198	0.197	
n^3D_1	0.308	0.223	
n^3D_3	1.311	1.035	
$1D_2$	1.125	0.852	
$1D'_2$	0.468	0.381	
$2D_2$	1.127	0.865	
$2D'_2$	0.466	0.368	
$3D_2$	1.128	0.871	
$3D'_2$	0.464	0.362	
n^3F_2	0.668	0.494	
n^3F_4	1.696	1.331	
$1F_3$	1.524	1.167	
$1F'_3$	0.827	0.646	
$2F_3$	1.524	1.169	
$2F'_3$	0.826	0.644	
$3F_3$	1.525		
$3F'_3$	0.825		
n^3G_3	1.040		
n^3G_5	2.081		
$1G_4$	1.918		
$1G'_4$	1.196		
$2G_4$	1.918		
$2G'_4$	1.195		

Table 1: Magnetic moments of B_c mesons states in the units of nuclear magneton μ_N.

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