Study of Thermodynamical Properties Through van der Waals HRG Model

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Introduction

Recent studies have explored the thermodynamics of hadronic matter using the van der Waals Hadron Resonance Gas (VDWHRG) model. This model incorporates attractive and repulsive interactions between hadrons, improving upon the ideal Hadron Resonance Gas model [3]. The VDWHRG model successfully describes lattice QCD data and predicts a liquid-gas phase transition at low temperatures and high baryon densities [2]. Studies have been investigated the effects of rotation [4] and extra resonance states on thermodynamic properties within this framework. The inclusion of van der Waals interactions leads to qualitatively different behavior of fluctuations of conserved charges in the crossover region, closely resembling lattice QCD results [3]. These studies highlight the importance of considering van der Waals interactions in hadronic systems and suggest that comparisons with ideal HRG models.

Model

In heavy-ion collisions, the fireball created can be effectively described by the grand canonical ensemble (GCE) [1], where the limited phase space probed by experiments allows for the conservation of energy, momentum, and charge to be neglected. The thermodynamic potential of the hadronic system is obtained from the contributions of all stable hadrons and known resonances. The partition function for the i^{th} particle species in the GCE of an ideal HRG is [1]:

$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ \ln\{1 \pm \exp[-(E_i - \mu_i)/T]\}, \quad (1)$$

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where g_i and $E_i = \sqrt{p^2 + m_i^2}$ are the degeneracy and energy of the i^{th} hadron, respectively. The $\pm$ sign corresponds to fermions and bosons. The chemical potential μ_i is given by [1]:

$$\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q, \quad (2)$$

where μ_B, μ_S and μ_Q are the baryon, strangeness and charge chemical potentials, respectively. B_i, S_i and Q_i are the baryon number, strangeness and electric charge of the i^{th} hadron.

The pressure P_i, energy density ε_i, number density n_i and entropy density s_i are obtained from the partition function as [1]:

$$P_i^{id} = \pm \frac{T g_i}{2\pi^2} \int_0^\infty p^2 dp \ \ln\{1 \pm \exp[-(E_i - \mu_i)/T]\} \quad (3)$$

$$\varepsilon_i^{id} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{E_i \ p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1} \quad (4)$$

$$n_i^{id} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1} \quad (5)$$

$$s_i^{id} = \pm \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp \Big[\ln\{1 \pm \exp[-(E_i - \mu_i)/T]\} \pm \frac{(E_i - \mu_i)/T}{\exp[(E_i - \mu_i)/T] \pm 1} \Big] \quad (6)$$

The van der Waals equation in the canonical ensemble is [2]:

$$\left(P + \left(\frac{N}{V}\right)^2 a\right)\left(V - Nb\right) = NT \quad (7)$$

where a and b are the VDW parameters describing attractive and repulsive interactions, respectively. P, V, T and N are the pressure, volume, temperature and number of particles.

In terms of number density $n \equiv N/V$, Eq.7 simplifies to [2, 3, 5]:

$$P(T, n) = \frac{nT}{1 - bn} - an^2, \tag{8}$$

where the repulsive term includes the hardcore radius r via $b = 16\pi r^3/3$.

The VDW equation of state in the GCE is :

$$P(T, \mu) = P^{id}(T, \mu^*) - an^2(T, \mu), \tag{9}$$

where the modified number density $n(T, \mu)$ is [2]:

$$n(T, \mu) = \frac{\sum_i n_i^{id}(T, \mu^*)}{1 + b \sum_i n_i^{id}(T, \mu^*)}. \tag{10}$$

Here, μ^* is the modified chemical potential :

$$\mu^* = \mu - bP(T, \mu) - abn^2(T, \mu) + 2an(T, \mu). \tag{11}$$

The entropy density $s(T, \mu)$ and energy density $\epsilon(T, \mu)$ are [2]:

$$s(T, \mu) = \frac{s^{id}(T, \mu^*)}{1 + bn^{id}(T, \mu^*)} \tag{12}$$

$$\epsilon(T, \mu) = \frac{\sum_i \epsilon_i^{id}(T, \mu^*)}{1 + b \sum_i n_i^{id}(T, \mu^*)} - an^2(T, \mu) \tag{13}$$

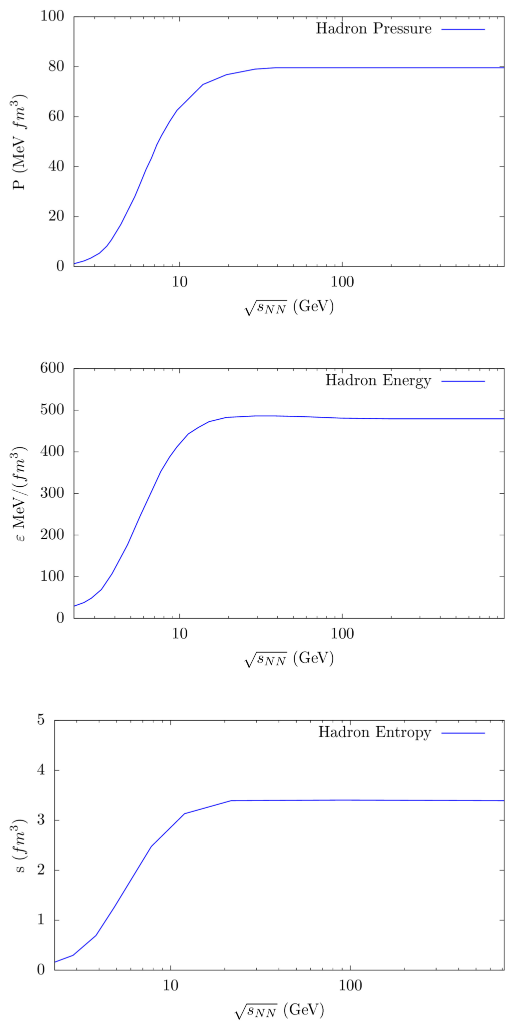
The VDWHRG model includes interactions only between baryons and anti-baryons, neglecting meson-meson and meson-(anti)baryon interactions [2].

Results and Conclusion

To study the variation of different thermodynamical quantities in ultrarelativistic heavy ion collisions we employ the following parameterization for the baryon chemical potential and temperature. [1, 5]:

$$\mu_B = \frac{c}{1 + d\sqrt{s_{NN}}}, \qquad T = e - f\mu_B^2 - g\mu_B^4 \tag{14}$$

we have explored the thermodynamic properties of hadrons using the van der Waals interacting model. We have successfully calculated the pressure, entropy, and energy density for a hadronic system, taking into account the effect of resonances. For mesons, we assumed a vanishing b parameter, while for the constant a, we used the previously established value $a = 329$ MeV fm^3 and an interaction radius $r = 0.59$ fm. We developed an ansatz for the chemical potential that incorporates freeze-out conditions and the influence of van der Waals interactions on the equation of state. Our analysis shows that these interactions lead to lower thermodynamic quantities than those observed in studies of ideal point-like and finite size interactions. By considering both attractive and repulsive interactions, we gain insights into hadronic matter behavior and its phase transitions in ultra-relativistic collisions.



Varation of Pressure, Energy density, Entropy density with $\sqrt{s}_{NN}$

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