Numerical solution of the radial Schrödinger equation with approximated screened potential

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Introduction

The exact solution of the radial Schrödinger equation for anharmonic potentials is a key topic in nuclear and particle physics, as it is essential for fully describing the quantum states of a system [1]. However, only a few anharmonic potentials allow for explicit solutions of the radial Schrödinger equation for all quantum numbers n and l.

The Nikiforov-Uvarov Functional Analysis (NUFA) method has proven to be a useful tool for solving the Schrödinger equation. Given the broad use of screened potentials to describe bound and continuum states in interacting systems, solving the non-relativistic radial Schrödinger equation for these potentials is of significant importance.

Theoretical Background

We are going to consider the potential describing a system with screened interactions

$$V(\boldsymbol{r}) = -\frac{4}{3}\frac{\alpha_s}{r} + \lambda\frac{(1-e^{-\delta r})}{\delta} + V_0 \quad (1)$$

The first term represents the attractive potential that diminishes with distance and second term represents an exponentially decreasing interaction. α_s is the strong coupling constant. δ is the screening parameter and λ the scaling parameter. The final term, V_0 shifts the potential by a constant reference value. This form is typical for models with both attractive and screened repulsive interactions. The equation (1) can be rewritten as:

$$V(\boldsymbol{r}) = -\frac{4}{3}\frac{\alpha_s}{r} + [\lambda r - \frac{\lambda\delta r^2}{2}] \quad (2)$$

The higher terms of r areneglected as their value have less significance in the potential [3]. This is in the in the form of the sextic and the Coulomb perturbed potentials. We use the NUFA method described in [2] to solve the potential.

We here are only solving the Coulomb perturbed potentials part of the solution. The N-dimensional radial Schrödinger equation for the Coulomb perturbed potentials are written as

$$\frac{d^2R(r)}{dr^2} + \frac{(N-1)}{r}\frac{dR(r)}{dr} + \frac{2\mu}{\hbar^2}[E + \frac{\lambda\delta r^2}{2} - \lambda r + \frac{4}{3}\frac{\alpha_s}{r} - \frac{l(l+N-2)\hbar^2}{2\mu r^2}]R(r) = 0 \quad (3)$$

Now by using the coordinate transformation $s = e^{-\alpha r}$, the Radial equation changes to,

$$\frac{d^2R}{ds^2} + \frac{1}{s}\frac{dR}{ds} + \frac{1}{s^2(1-s)}[(\frac{2\mu E}{\hbar^2\alpha^2} - \frac{6\mu\lambda\delta^2}{\hbar^2\alpha^4} - \frac{6\mu\lambda}{\hbar^2\alpha^3})s^2 + (\frac{4\mu\lambda\delta^2}{\hbar^2\alpha^4} - \frac{4\mu E}{\hbar^2\alpha^2} + \frac{6\mu\lambda}{\hbar^2\alpha^3} - \frac{8\mu\alpha_s}{3\hbar^2\alpha})s + (\frac{2\mu E}{\hbar^2\alpha^2} - \frac{\mu\lambda\delta^2}{\hbar^2\alpha^4} - \frac{2\mu\lambda}{\hbar^2\alpha^3} + \frac{8\mu\alpha_s}{3\hbar^2\alpha} - l(l+N-2))]R(s) = 0$$

Comparing with the NU equation in [2] we get the following parametric equations,

$$\epsilon = -\frac{2\mu E}{\hbar^2\alpha^2} \quad (4)$$

$$\tau_1 = -\frac{6\mu\lambda\delta}{\hbar^2\alpha^4} + \frac{6\mu\lambda}{\hbar^2\alpha^3} \quad (5)$$

$$\tau_2 = \frac{4\mu\lambda\delta^2}{\hbar^2\alpha^4} + \frac{6\mu\lambda}{\hbar^2\alpha^3} - \frac{8\mu\alpha_s}{3\hbar^2\alpha} \quad (6)$$

$$\tau_3 = \frac{\mu\lambda\delta^2}{\hbar^2\alpha^4} + \frac{2\mu\lambda}{\hbar^2\alpha^3} - \frac{4\mu\alpha_s}{3\hbar^2\alpha} + l(l+N-2) \quad (7)$$

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By commparing (4) to (7) we can find the following exponent.

$$\gamma = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{12\mu\lambda\delta}{\hbar^2\alpha^4} + \frac{8\mu\lambda}{\hbar^2\alpha^3} + 4l(l+N-2)}$$

Energy eigen value is given by,

$$E = \frac{-\hbar^2\alpha^2}{2\mu}[\frac{-(\gamma+n)^2+\tau_1-\tau_3}{2(\gamma+n)}]^2 + \frac{\hbar^2\alpha^2\tau_3}{2\mu} \tag{8}$$

Results and Discussions

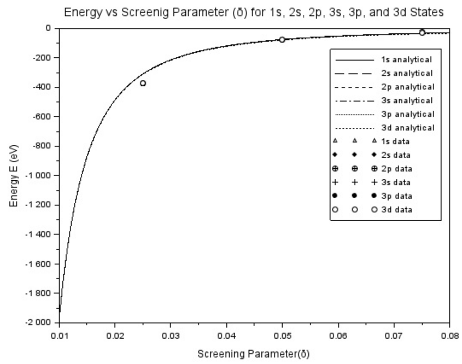

FIG. 1: Variation of Energy vs screening parameter δ.

TABLE I: Energy values for different states

State	α	E (eV)
1s	0.025	-379.9985
	0.075	-25.91327
2s	0.025	-371.99
	0.075	-24.40
2p	0.025	-373.06
	0.075	-25.47
3s	0.025	-368.56
	0.075	-23.74
3p	0.025	-369.70
	0.075	-24.88
3d	0.025	-371.99
	0.075	-27.16

In this work we have approximated the screened potential using Taylor series and

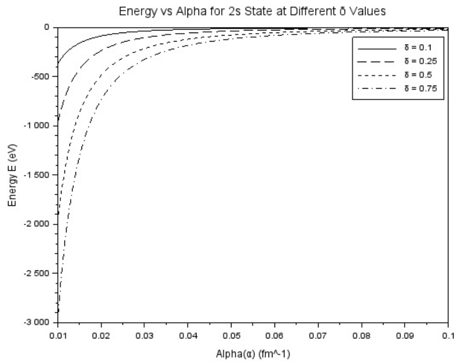

FIG. 2: Variation of Energy vs α for various values of δ

formed upto a sextic potential term. We have used α, λ, δ as the parameters. We have taken the value $\lambda = 0.1$. Fig. 1 shows the plot of Energy E vs the screening parameter δ for various l values. Fig. 2 shows the plot of Energy E vs α for various values of δ. The above plots gives the relation between the energy, screening parameter and α for a given value of λ.

The results of the present work can be used to evaluate mass spectra of hadrons using the relation $M = (\sum_i m_i) + E$ where m_i gives the quark masses [1].

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