Effect of dimensionality number on thermodynamic properties of heavy quarkonia

Rishabh Sharma[1*] and Vineet Kumar Agotiya[1]
[1]Department of Physics, Central University of Jharkhand, Ranchi - 835222, INDIA

Introduction

In high-energy physics, thermodynamic properties are very crucial for understanding quark-gluon plasma and heavy quarkonium systems (charmonium and bottomonium). In this article we find the thermodynamic properties [1] of ground state of Charmonium using the partition function approach by solving the Schrodinger equation for Hellmann-plus-Screened-Kratzer (HSK) potential with Nikiforov-Uvarov Functional Analysis (NUFA) method [2]. NUFA Method is a modification in the NU method and it simplifies some of the mathematical complexities left behind in the NU method [3]. This method is derived by combination of the Parametric NU method and the Factorization method.

In this work we have used the combined form of Hellmann and Screened Kratzer potential [4] reads,

$$V(r) = -\frac{Ae^{-m_D(T)r}}{r} + \frac{Be^{-m_D(T)r}}{r^2} - \frac{C}{r} + \frac{De^{-m_D(T)r}}{r} \tag{1}$$

Where A, B, C and D are some potential parameters those numerical values are taken as, $A = 0.52$, $B = 3.9422$, $C = 0.722$, and $D = 0.23$ respectively and ϵ is a screening parameter. $m_D(T)$ is the quasi particle (QP) Debye mass [5], Debye mass in terms of temperature and magnetic field representing non-interacting quarks and gluons is given as

$$m_D^2\left(T, eB\right) = 4\pi\alpha_s\left(T^2 + \frac{3.eB}{2\pi^2}\right) \tag{2}$$

Here α_s(T) is two loop running coupling constant whose value is as follows;

$$\alpha_s\left(T\right) = \frac{6\pi}{\left(33 - 2N_f\right)\phi}\left(1 - A\frac{log2\phi}{\phi}\right) \tag{3}$$

where, $\phi = log\left(\frac{T}{\lambda_T}\right)$ and A $= 3 * (153 - 19N_f)/(33 - 2N_f)^2$ and λ_T is QCD coupling scale and its value is 0.197 GeV.

Methodology

The N-dimensional Schrodinger equation [3] for two interacting quarks in a quarkonium state is given by,

$$\frac{d^2R(r)}{dr^2} + \left[\frac{\left(l + \frac{N-2}{2}\right)^2 - \frac{1}{4}}{r^2} + 2\mu\left(E - V(r)\right)\right]R(r) = 0 \tag{4}$$

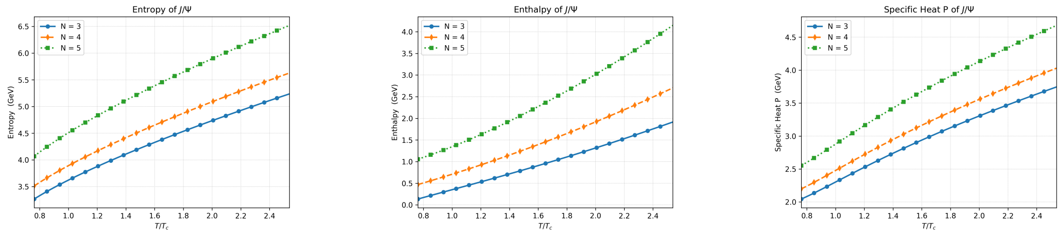
Where l, μ, and 'r' are the angular momentum quantum number, reduced mass of heavy quarkonium system, and interparticle distance respectively. On solving and comparing the above equation with parametric form NU method we get the expression of binding energy,

$$E = -\frac{Pm_D^2(T)}{2\mu} - Cm_D(T) - \frac{m_D^2(T)}{8\mu}\left[\frac{1}{\zeta}\left(P + \frac{2\mu}{m_D(T)}(A + C - D)\right) - \zeta\right]^2 \tag{5}$$

The partition function for the quarkonium states is given by,

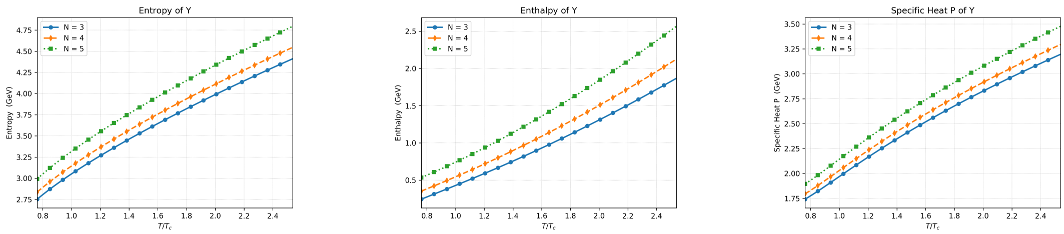
$$Z = \sum_i e^{-\beta E_i} \tag{6}$$

*Electronic address: sharmacuj@gmail.com



(a) (b) (c)

FIG. 1: This figure represent the variation of entropy in (a), enthaphy in (b) and specific heat in (c) for J/ψ with T/T_C for different values of dimensionality number (N= 3, 4, 5) at fix value of magnetic field eB= 0.5 GeV^2.



(a) (b) (c)

FIG. 2: This figure represent the variation of entropy in (a), enthaphy in (b) and specific heat in (c) for Υ with T/T_C for different values of dimensionality number (N= 3, 4, 5) at fix value of magnetic field eB= 0.5 GeV^2.

where, $\beta = \dfrac{1}{k_B T}$ and k_B is the Boltzmann's constant and 'T' is the temperature of the system. The internal energy (U) is given by

$$U = -\frac{\partial \ln Z}{\partial \beta} \tag{7}$$

Results and Conclusion

In this study, we investigated the thermodynamic behavior of the ground states of heavy quarkonia, specifically J/ψ and Υ. Figures 1 and 2 illustrate the variations in entropy, enthalpy, and specific heat as functions of T/T_c for different dimensionality numbers (N=3, 4, 5) for J/ψ and Υ, respectively. Our findings indicate that as the dimensionality factor increases, all three thermodynamic parameters (entropy, enthalpy, and specific heat) also increase with temperature. This type of studies provides deeper insights about fundamental interactions within higher-dimensional frameworks, also contribute to our understanding of conditions in the early universe.

References

[1] S. Solanki et al., Int. Jour. modern phys. A **37**, 2250196 (2022).

[2] E.P. Inyang, E.S. William, J.E. Ntibi, J.A. Obu, P.C. Iwuji, E.P. Inyang, Canadian Journal of Physics **100**, 463 (2022).

[3] R. Sharma et al.,Int. Jour. modern phys. A **39**, 2450071 (2024).

[4] A.N. Ikot, U.S. Okorie, R. Sever, G.J. Rampho, The European Physical Journal Plus **134**, 386 (2019).

[5] M. Lal et al., Indian Jour. Pure Applied Phys. **60**, 475 (2022).