Bulk Viscosity near QCD phase transition and Spontaneous Chiral Symmetry Breaking Effect

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We consider a strongly interacting fermionic matter that is in a state with spontaneously broken chiral symmetry. Invoking quasi-particle description for strongly interacting fermions with dynamic quasi-particle excitations we evaluate the thermodynamic properties of this matter in relativistic mean-field approximation. Taking into consideration Boltzmann transport equations in relaxation time approximation we evaluate the transport properties of this medium. It is found that due to the coupling of fermionic quasi-particle excitations and σ modes (whose strength is governed by the expectation value of Chiral Condensate in this model) ζ/s can get singular near transition region both along $O(4)$ transition line and in $Z(2)$ universality class. Our results remain applicable in the limit of vanishing expectation value of Chiral Condensate that is in the limit $T \sim T_C$, where σ field strength is given by its vacuum expectation value.

1. Introduction

The dynamics of strongly interacting matter can be evaluated by estimating the transport properties of the system under consideration. These properties are encoded in shear and bulk viscosity of the medium and are of immense significance for understanding the nature of QCD matter. The behaviour of these transport coefficients can provide important information regarding the dynamics of the strongly interacting matter and importantly about their dynamics near region of phase transition [1–3]. In case of Hadron Resonance Gas model it has been found that ζ/s becomes singular near transition region due to the effect of Hagedorn states. For $SU(3)$ gluodynamics, bulk viscosity can get singular near transition region due to fast growth of thermal expecatation value of trace of energy momentum tensor. A similar result invoking Kubo formalism, low energy theorems of QCD and Lattice data was reported near critical region of phase transition. In this article we utilize Linear Sigma model for fermionic matter in a state with broken continious symmetry and arrive at a decription in terms of dynamic quasiparticle fermionic excitations that are coupled to σ and $\vec{\pi}$ modes. We show that singular behaviour of ζ/s near transition region is a general result and arises due to the coupling of fermionic quasiparticle excitations with the σ modes that are generated due to spontaneous chiral symmetry breaking in case of QCD phase transition.

MODEL: Let us consider a Linear Sigma model in presence of arbitrary interactions among fermions

$$\mathcal{L} = \bar{\psi}\,(iD)\,\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + g\bar{\psi}\psi\sigma + ig\bar{\psi}\vec{\tau}\gamma_5\psi\cdot\vec{\pi} - \frac{\mu^2}{2}(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{24}(\sigma^2+\vec{\pi}^2)^2 + \frac{1}{4}X_{\mu\nu}X^{\mu\nu}, \quad (1)$$

where ψ is fermionic field, $D \equiv \gamma^\mu D_\mu$, with covariant derivative $D_\mu = \partial_\mu - ie\chi_\mu$, e is the charge on fermion and χ_μ is the four-potential of exchange field. μ^2 is the mass term and

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g, λ are coupling constants. The pseudoscalar field $\vec{\pi} \equiv (\pi_1, \pi_2, \pi_3)$ is an isotriplet and in this case corresponds to massless pion field. σ is an isosinglet field. $X_{\mu\nu} = (\partial_\mu \chi_\nu - \partial_\nu \chi_\mu)$ is the field strength tensor.

This model exhibits spontaneous symmetry breaking and classical ground state is determined by the potential that describes self interactions of scalars

$$V(\sigma, \vec{\pi}^2) = \frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{24}(\sigma^2 + \vec{\pi}^2)^2. \quad (2)$$

For $\mu^2 < 0$, the minima of this potential is located at $\sigma^2 + \vec{\pi}^2 = -\frac{6\mu^2}{\lambda} = v^2$, which is its vacuum expectation value. Expanding the σ field around a particular ground state $\langle\sigma\rangle = v$ and $\langle\vec{\pi}\rangle = 0$ as $\sigma = v + \sigma'$, one can rewrite the Lagrangian (1) as

$$\begin{aligned}\mathcal{L} = &\bar{\psi}(iD - m)\psi + \frac{1}{2}(\partial_\mu \sigma')^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 \\ &+ \mu^2 \sigma'^2 - \frac{\lambda}{6} v \sigma' (\sigma'^2 + \vec{\pi}^2) - \frac{\lambda}{24}(\sigma'^2 + \vec{\pi}^2)^2 \\ &+ ig\bar{\psi}\vec{\tau}\gamma_5 \psi \cdot \vec{\pi} + g\bar{\psi}\psi\sigma' + \frac{1}{4} X_{\mu\nu} X^{\mu\nu},\end{aligned} \quad (3)$$

where $M = gv$ is the mass acquired by fermions. Invoking quasiparticle description for fermions and using relativistic mean-field approximation for σ and $\vec{\pi}$ modes that represent long range residual interaction,

$$\begin{aligned}\sigma(\vec{x}, t) &\to \langle\sigma(\vec{x}, t)\rangle = \sigma_0 \\ \vec{\pi}(\vec{x}, t) &\to \langle\vec{\pi}(\vec{x}, t)\rangle = 0,\end{aligned}$$

the variation in energy momentum tensor $\delta T_{\mu\nu}$ becomes

$$\begin{aligned}\delta T_{\mu\nu} = &\int d\Gamma \frac{p_\mu p_\nu}{\sqrt{p^2 + M^{*2}}}(\delta f + \delta \bar{f}) \\ &-\eta_{\mu\nu} g \sigma_0 \int d\Gamma \frac{M^*}{2\sqrt{p^2 + M^{*2}}}(\delta f + \delta \bar{f}),\end{aligned} \quad (4)$$

where $M^* = M - g\sigma$ and $M = M(T, \mu)$ is the dynamic mass of fermionic quasiparticle excitation and $\eta_{\mu\nu}$ is the metric. Now using relativistic Boltzmann transport equation

$$p^\mu \partial_\mu f = -\frac{p^\mu \cdot u_\mu}{\tau} \delta f, \quad (5)$$

the bulk and shear viscosity becomes

$$\begin{aligned}\zeta = &-\int d\Gamma \frac{1}{ET}\left(\frac{1}{3}M^{*2} - \frac{1}{2} g \sigma_0 M^*\right) \\ &\times \Big\{ \left[\tau f_0(1 - f_0) + \bar{\tau}\bar{f}_0(1 - \bar{f}_0)\right] \\ &\times \left(\frac{\vec{p}^2}{3E} - \left(\left(\frac{\partial P}{\partial \epsilon}\right)_n \left(E - T\frac{\partial E}{\partial T} - \mu \frac{\partial E}{\partial \mu}\right) - \left(\frac{\partial P}{\partial n}\right)_\epsilon \left(\frac{\partial E}{\partial \mu}\right)\right)\right) \\ &-\frac{M^{*2}}{3E}\left(\tau f_0(1 - f_0) - \bar{\tau}\bar{f}_0(1 - \bar{f}_0)\right)\left(\frac{\partial P}{\partial n}\right)_\epsilon \Big\}\end{aligned} \quad (6)$$

$$\eta = \frac{1}{15}\int d\Gamma \left(\frac{1}{E^2 T}\right)\vec{p}^4 \times \left[\tau f_0(1 - f_0) + \bar{\tau}\bar{f}_0(1 - \bar{f}_0)\right] \quad (7)$$

Scaling in $O(4)$ and $Z(2)$ universality class:

Along $O(4)$ transition line the singular part of free energy is

$$F_s(T, \mu) = (-t)^{2-\alpha} f_s(1, (-t)^{\beta\delta}(-h)), \quad (8)$$

which leads to

$$\zeta/s \sim (-t)^{\alpha + 4\beta - 2}. \quad (9)$$

For $Z(2)$ universality class the free energy is

$$F_s(T, \mu) \sim (-h)^{1+\frac{1}{\delta}} f_s((-t)(-h)^{-1/\beta\delta}, 1), \quad (10)$$

which leads to

$$\zeta/s \sim (-h)^{\gamma/\beta\delta + 4/\delta - 2}. \quad (11)$$

For static critical exponents $\alpha = -0.24$, $\beta = 0.38$ in case of $O(4)$ and $\beta = 0.31$, $\delta = 5.2$ and $\gamma = 1.52$ in case of $Z(2)$ universality class ζ/s becomes singular near transition region.

References

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